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Scaling, self-similarity and multifractality in FX markets

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Abstract

This paper presents an empirical investigation of scaling and multifractal properties of US Dollar–Deutschemark (USD–DEM) returns. The data set is ten years of 5-min returns. The cumulative return distributions of positive and negative tails at different time intervals are linear in the double logarithmic space. This presents strong evidence that the USD–DEM returns exhibit power-law scaling in the tails. To test the multifractal properties of USD–DEM returns, the mean moment of the absolute returns as a function of time intervals is plotted for different powers of absolute returns. These moments show different slopes for these powers of absolute returns. The nonlinearity of the scaling exponent indicates that the returns are multifractal.

JEL classification: G0; G1; C1

Keywords: Scaling; Self-similarity; Multifractality; High-frequency data; Foreign exchange markets

1. Introduction

Researchers have been investigating scaling laws in finance for a long time. The beginnings may be traced back to the late 1920s.[1] At that time, the work emphasized

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[1] In the late 1920s, Elliot wave principle claimed that stock markets do not behave in a chaotic manner, but move in repetitive cycles.

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the appearance of patterns at different time scales. In the 1960s, a class of stable distributions was put forward to account for the power-law tail behavior of financial series. [2] Fractals and chaos coming from physical science led to a new wave of interest in scaling in the 1980s. [3] In recent years, the study of scaling laws resurged due to the availability of high-frequency data.

Scaling expresses invariance with respect to translation in time and change in the unit of time. That is, except for amplitude and rate of change, the rules of higher- and lower-frequency variation are the same as the rules of mid-speed frequency variation, Mandelbrot [4]. Scaling is a rule that relates returns over different sampling intervals. The shape of the distribution of returns should be the same when the time scale is changed, Calvet and Fisher [5]. In empirical studies, the scaling analysis typically exploits some kind of linear relationship between logs of variables.

In the literature, many empirical studies have shown that financial time series exhibit scaling like characteristics. Müller et al. [6] and Guillaume et al. [7] reported an empirical scaling law for mean absolute price changes over a time interval for foreign exchange rate. Dacorogna et al. [8] presented empirical scaling laws for US Dollar–Japanese Yen (USD–JPY) and British Pound–US Dollar (GBP–USD). Mantegna and Stanley [9] also found scaling behavior in the Standard and Poor index (S&P 500) by examining high frequency data. Recently, Gençay et al. [10] suggested that financial time series may not follow a single-scaling law across all horizons. They used a wavelet multi-scaling approach to show that foreign exchange rate volatilities follow different scaling laws at different horizons. They provided evidence that there was no unique global scaling in financial time series but rather scaling was time varying.

However, some literature continued to question the evidence of the scaling laws in foreign exchange (FX) markets. LeBaron [11] examined the theoretical foundation of scaling laws. He demonstrated that many graphical scaling results could have been generated by a simple stochastic volatility model. He suggested that dependence in the financial time series might be the key cause in the apparent scaling observed. LeBaron [12] presented a simple stochastic volatility model, which was able to produce visual power-laws and long memory similar to those from actual return series using comparable sample sizes. However, Stanley et al. [13] pointed out that a three-factor model cannot generate power-law behavior.

Whether or not the financial time series follow power-law and the type of scaling rule they obey are still open questions. In this paper, we will investigate intra-day US Dollar–Deutschemark (USD–DEM) returns and provide evidence that the tails of returns do follow power-law. Furthermore, the returns exhibit multifractal behavior. Section 2 is on the discussion of two types of scaling behaviors of USD–DEM returns. Namely, the behavior of the tails of the distribution of returns keeping the time interval of returns constant and the behavior of the moments of the absolute value of returns as a function of time interval. We conclude afterwards.

[2] See Mandelbrot [1] and Fama [2] for details.

[3] See Peters [3] for a survey of fractals and chaos.