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# Indeterminacy in foreign exchange markets

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**Abstract**

We discuss price variations distributions in foreign exchange markets, characterizing them both in calendar and business time frameworks. The price dynamics is found to be the result of two distinct processes, a multi-variance diffusion and an error process. The presence of the latter, which dominates at short time scales, leads to indeterminacy principle in finance. Furthermore, dynamics does not allow for a scheme based on independent probability distributions, since volatility exhibits a strong correlation even at the shortest time scales. 

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*Keywords:* Foreign exchange markets; High-frequency data; Indeterminacy principle; Multi-variance process; Error process

## 1. Introduction and definition of time

The characterization of price dynamics in financial markets is an old but still puzzling problem. At the beginning of the century, Bachelier [1] proposed to consider price variations as independent realizations of identically Gaussian distributed variables, while in the 1963 Mandelbrot [2] introduced symmetric Lévy stable distributions. In the last years, Mantegna and Stanley [3] (and see also Refs. [4,5]) provided evidence that a Lévy stable process well reproduces the central part of high-frequency price variation distribution, while the tails are approximately exponential. The Bachelier's Gaussian shape is recovered only on longer time scales which are, typically, of order one month. A common point of [1–5] is that price dynamics is considered as the result

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of independent random variables. This kind of approach seems inadequate, since there is evidence of volatility correlations on long run [6–13].

Indeed, price distribution strongly depends on how one measures time flow. The choice of time index is twofold, calendar time and business time. Business time is the sequence of integers $n = 1, 2, 3, \ldots$ which indexes successive established quotes. These integers correspond, respectively, to the calendar times $t_1, t_2, \ldots$. Therefore, calendar time (which is monotonically increasing) is a stochastic process of business time. The relation can be inverted by considering $n$ as a function of calendar time, i.e., $n = n_t$, but in this case the function is defined only on the sequence of quoting calendar times. The price dynamics, therefore, can be described with respect to business time by means of series $S_n$ or with respect to the calendar time by means of $S_t \equiv S_{n_t}$. In the first cases quotes $S$ are defined on all integers and lags are all equal, in the second, they are defined only on quoting times and lags are unequal.

The main result of this paper is that a price quote is due to two distinct independent stochastic processes: an *error* process superposed to an *underlying* price process. The latter evolves following calendar time, while the former is due to an erroneous evaluation of a market operator, and its natural frequency is marked by business time. The resulting price variation distribution is, therefore, the convolution of two distributions associated to these two distinct processes: the error process distribution, which does not change with time, and the distribution of the underlying process, which scales, on the contrary, with calendar time.

The error process produces always a gap between two consecutive price estimations even if they are almost contemporary. This phenomenology strongly reminds quantum mechanics, where a measurement result always has a minimum uncertainty as stated by the Heisenberg Principle. Following this comparison, one can state an *indeterminacy principle* in finance: a price is never given with a precision less than a natural constant for that market.

Another important fact which is due to this phenomenology is that two consecutive price variations cannot be considered fully independent random variables, but they exhibit a very peculiar anticorrelation as we will see. Moreover, we provide evidence that volatility is so strongly correlated that remains substantially constant inside the largest lag we consider, and therefore the usual Lévy stable scheme seems to be not appropriate.

In this paper we examine the high-frequency price variation distribution of three foreign exchange markets, the Deutsch Mark/US Dollar (DEM-USD) exchange in 1993 (1,472,140 quotes) and in 1998 (1,620,843 quotes), the Japanese Yen/US Dollar (JPY-USD) in 1993 (570,713 quotes), and the Japanese Yen/Deutsch Mark (JPY-DEM) in 1993 (158,878 quotes). The quotes represent the value of one US Dollar in Deutsch Marks and in Japanese Yens respectively, the DEM-USD and JPY-USD cases, while they represent the value of one Deutsch Mark in Japanese Yens in the DEM-JPY case. The price changes are given in pips, which indicate a DEM/10,000 in the DEM-USD case, and a Yen/100 in the JPY-USD and JPY-DEM cases. All the data sets analyzed in this work have been provided by Olsen & Associates.