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Research paper

Dynamics of a developing economy with a remote region: Agglomeration, trade integration and trade patterns

Pasquale Commendatore[a,*], Ingrid Kubin[b], Iryna Sushko[c]

[a] Dept. of Law, University of Naples 'Federico II', Italy
[b] Dept. of Economics, Vienna University of Economics and Business Administration, Austria
[c] Institute of Mathematics, National Academy of Sciences of Ukraine, Ukraine

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ABSTRACT

We consider a three-region developing economy with poor transport infrastructures. Two models are related to different stages of development: in the first all regions are autarkic; in the second two of the regions begin to integrate with the third region still not accessible to trade. The properties of the two models are studied also considering the interplay between industry location and trade patterns. Dynamics of these models are described by two-dimensional piecewise smooth maps, characterized by multistability and complex bifurcation structure of the parameter space. We obtain analytical results related to stability of various fixed points and illustrate several bifurcation structures by means of two-dimensional bifurcation diagrams and basins of coexisting attractors.

1. Introduction

We model a three-region economy representing a less-developed Country with weak transport infrastructures. Due to this, it is difficult to trade goods across regions (for an analogous interpretation see [2] in the context of a two-region economy with non-traded goods). We assume a specific geography: two regions are relatively close to each other, whereas the third one is remote and difficult to access. We consider two stages of development: in the first stage costs of trading goods are sufficiently high that none of the regions trade, in the second stage the first two regions improve their integration by lowering trade costs. Depending on the degree of integration and on the distribution across space of the industrial activity, different trade patters may emerge: absent, unilateral or bilateral trade.

We study the existence and local stability properties of the possible long-run equilibria and the global stability properties for the two corresponding models, the dynamics of which are described by two-dimensional piecewise smooth maps depending on nine parameters. Considering Model 1 we focus on the influence on the asymptotic behavior of trajectories of the values of parameters L and E related to the local market size and the overall number of entrepreneurs, respectively. The dynamics of the map, characterized by multistability, for increasing L evolves from coexisting attracting Core-periphery fixed points to several coexisting chaotic attractors. However, for large values of L the Core-periphery fixed points, being locally repelling, become attracting in Milnor sense.[1] The dynamics of the map which is related to Model 2 is much more

* Corresponding author.
E-mail address: commenda@unina.it (P. Commendatore).

[1] Recall that according to the most spread definition, an attractor is a closed invariant set with a dense orbit, which has a neighborhood *each point* of which is attracted to the attractor, while an attractor in Milnor sense [9] does not require the existence of such a neighborhood, but only a *positive measure*

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complicated. We describe several bifurcation scenarios observed in the system for increasing value of parameter ε related to the trade costs. Besides multistability, a characteristic property of this map is associated with border collision bifurcations[2] (BCB for short) which an attractor may undergo colliding with the borders between unilateral, bilateral and no trade phase regions. In particular, we present examples of a fold BCB at which attracting and saddle fixed points are born not due to an eigenvalue equal to 1 (as in case of a 'smooth' fold bifurcation) but due to a collision with the border. An example of a BCB from an attracting 2-cycle to an attracting 6-cycle is also presented which has no smooth analogues. In fact, bifurcation structure of the parameter space of Model 2 associated with attracting cycles and chaotic attractors is very complicated and required an additional study.

We briefly compare our results concerning the equilibrium properties of a three-region linear new economic geography (NEG) model with the two-region counterparts developed by in Behrens [2] and [3] in the absence of a third remote region. We also clarify the relationship between agglomeration and emerging trade patterns.

The structure of the paper is the following. Section 2 lays down the economic framework, introducing the two different models representing two stages of development. In Section 3, we discuss dynamic properties of Model 1, in particular, for Core-periphery, border symmetric/asymmetric and interior symmetric/asymmetric fixed points we obtain stability conditions and describe bifurcations which these fixed points can undergo. The basic bifurcation scenario observed in the system is illustrated by means of two-dimensional bifurcation diagrams and examples of basins of coexisting attractors. In Section 4, we obtain some analytical and numerical results related to Model 2. In particular, examples of border collision bifurcations, which enrich the observed bifurcation scenarios, are presented. In Section 5, we discuss the obtained results.

2. The economic framework

2.1. General set-up

The economy is composed of three regions (labeled $r = 1, 2, 3$); two sectors, agriculture (A) and manufacturing (M); and two factors of production, entrepreneurs (E) and workers (L). In the perfectly competitive A-sector it is produced an homogeneous good, whereas in the monopolistically competitive M-sector, the output is represented by N varieties of a differentiated good. Workers can move between sectors but are interregionally immobile. We assume that they are equally distributed across regions, i.e. $L_1 = L_2 = L_3 = \frac{L}{3}$. Entrepreneurs, which are specific to the M-sector, are allowed to migrate.

In the A-sector, the production of 1 unit of output requires 1 unit of labor. In the M-sector, instead, a fixed component necessarily enters in the production of a variety i, corresponding to 1 entrepreneur. Moreover, for each additional unit of the variety i, η units of labor enter in its production. Thus, total costs (TC) can be expressed as:

$$TC = \pi_i + w\eta q_i$$

where π_i represents the operating profits and the remuneration of the entrepreneur, w the wage rate, η the labor input requirement and q_i the quantity produced of variety i.

Due to increasing returns and in the absence of economies of scope, each firm produces a single variety. Following our assumption that only one entrepreneur is required to activate production, the number of firms (N) is equal to the number of varieties and to that of entrepreneurs. Denoting by λ_r the share of entrepreneurs located in region r, the number of regional varieties produced in that region is:

$$n_r = \lambda_r N = \lambda_r E$$

where $r = 1, 2, 3$.

The cost of trading goods from r to s (or in the opposite direction from s to r) is $T_{rs} = T_{sr} > 0$ for $r \neq s$; and $T_{rr} = 0$ for $r, s = 1, 2, 3$. At the beginning, we assume that trade costs are sufficiently large that all the regions are in autarky (no M goods are traded across the regions). We set $T_{12} = T > 0$, where T (to be fixed below) is sufficiently large to ensure that regions 1 and 2 do not trade with each other; we also set $T_{13} = T_{23} = T_A > T$, which implies that region 3 is the more distant region in terms of trade costs. Because of that also region 3 does not trade with the other two. This set-up generalizes [2], who considers the case of two regions in full autarky. Then, we study the case when starting from the previous situation, we reduce trade costs between regions 1 and 2. We set $T_{12} = T - \varepsilon$, with $0 < \varepsilon < T$, then we reduce ε allowing for trade between 1 and 2. Since trade costs between regions 1 and 3 as well as 2 and 3 are the same as before, region 3 still does not trade with the other two. In this second set-up: $T_{13} = T_{23} = T_A > T_{12} = T - \varepsilon > 0$.

The representative consumer's (unskilled worker or entrepreneur) utility function is quasi-linear (see [11]), composed of a quadratic part defining preferences across the N varieties of the M-good and a linear component for the consumption of

set of points attracted to the attractor. For short, we say that a closed invariant set is an *M-attractor* if it is attracting in Milnor sense, but it does not have an attracting neighborhood.

[2] Recall that the term 'border collision bifurcation' was introduced in [10] to denote a qualitative change of the dynamics of a nonsmooth map observed when its invariant set, e.g., a fixed point, collides with a border along which the system function changes its definition.

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