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# Efficiency and surplus bounds in Cournot competition

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**Abstract**

We derive bounds on the ratios of deadweight loss and consumer surplus to producer surplus under Cournot competition. To do so, we introduce a parameterization of the degree of curvature of market demand using the parallel concepts of $\rho$-concavity and $\rho$-convexity. The "more concave" is demand, the larger the share of producer surplus in overall surplus, the smaller is consumer surplus relative to producer surplus, and the lower the ratio of deadweight loss to producer surplus. Deadweight loss over total potential surplus is at first increasing with demand concavity, then eventually decreasing.


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## 1. Introduction

Imperfect competition distorts market allocations by raising the equilibrium price above marginal cost. The size of the distortion depends upon the industry demand curve and the number of competing firms. We quantify this distortion according to various surplus benchmarks, as a function of the number of competitors and the curvature of the demand curve for Cournot interaction. We show that the fraction of potential (first-best) social surplus captured by producers increases as demand becomes more concave. We also provide bounds on consumer surplus and

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deadweight loss which depend on (potentially) observable magnitudes, such as producer surplus. These bounds depend on two parameters that measure the generalized concavity and convexity of demand.

The paper complements three bodies of literature on imperfect competition. The first addresses market performance under imperfect competition, and traces its lineage back through Mankiw and Whinston [9], through Spence [12] and Dixit and Stiglitz [7], and ultimately to Chamberlin [3]. The emphasis has been on the long-run equilibrium, with the number of firms used to measure market performance, but there has been no attempt to quantify deadweight loss. By contrast, our work is a short-run analysis, with the number of firms fixed. We consider the size of the various surpluses reaped (producer surplus and consumer surplus) and unreaped (deadweight loss) in the market.

The second literature concerns estimation of welfare loss due to market power, and goes back to Harberger's [8] provocative study that estimated monopoly deadweight loss as 0.1% of GNP. This famous study of distortionary "triangles" has been criticized in several respects, including the use of the profit data, the assumptions of linear demand and unit elasticity of demand for all industries. Subsequent studies (also criticized heavily) have used profit and cost data differently, and typically have assumed linear demand or a constant elasticity. Cowling and Mueller [4] have suggested that welfare loss could be up to 14% of GNP. We do not further investigate the use of profit data, but we do specify a consistent theoretical model that starts with the equilibrium oligopoly pricing condition and uses it to derive bounds on deadweight loss that depend on the curvature of demand.

The third complementary body of literature uses extended concavity concepts to establish equilibrium existence and uniqueness in the Cournot model. This literature goes back through Novshek [11] to McManus [10]. Most recently, Deneckere and Kovenock [6] have synthesized previous results and recast them in terms of demand properties.

The present analysis uses the concept of $\rho$-concavity that was introduced into economics by Caplin and Nalebuff [1] and applied to (Bertrand) oligopoly in Caplin and Nalebuff [2]. The larger is $\rho$, the "more concave" the demand function. To obtain a tighter characterization of demand curvature we also use the parallel concept of $\rho$-convexity whereby the lower $\rho$ the "more convex" is demand.

Section 2 presents a general background to $\rho$-concavity and $\rho$-convexity and delivers relations between functions and their inverses. Section 3 constitutes the core of the paper. For $n$ firms in a Cournot oligopoly and an observed equilibrium price and quantity, we first determine bounds on the actual demand curve given that it must lie between two curvature bounds. These bounds on the demand function then determine the bounds on several surplus measures, such as consumer surplus, deadweight loss, and the fraction of producer surplus in the total potential surplus (perfectly competitive benchmark). Ratio forms (and often tighter bounds) are given for the symmetric cost case, and intuition is then provided for $\rho$-linear demands. Section 4 concludes with comments on the welfare costs of excessive entry.