Computers & Operations Research 38 (2011) 1367–1376

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Bee colony optimization for the *p*-center problem ☆

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A R T I C L E I N F O

Available online 17 December 2010

Keywords:
Meta-heuristics
Bee colony optimization
Combinatorial optimization
Location analysis
p-Center problem

A B S T R A C T

Bee colony optimization (BCO) is a relatively new meta-heuristic designed to deal with hard combinatorial optimization problems. It is biologically inspired method that explores collective intelligence applied by the honey bees during nectar collecting process. In this paper we apply BCO to the *p*-center problem in the case of symmetric distance matrix. On the contrary to the constructive variant of the BCO algorithm used in recent literature, we propose variant of BCO based on the improvement concept (BCOi). The BCOi has not been significantly used in the relevant BCO literature so far. In this paper it is proved that BCOi can be a very useful concept for solving difficult combinatorial problems. The numerical experiments performed on well-known benchmark problems show that the BCOi is competitive with other methods and it can generate high-quality solutions within negligible CPU times.

1. Introduction

Meta-heuristics have become very powerful tool for solving hard combinatorial optimization problems. Among them a class of biologically inspired algorithms can be recognized. Bee colony optimization (BCO) method, that explores collective intelligence applied by the honey bees during nectar collecting process, is one of them. BCO has been proposed by Lučić and Teodorović [1–3] and up to now it is successfully applied to various real-life optimization problems.

The BCO is a stochastic, random-search technique that belongs to the class of population-based algorithms. This technique uses an analogy between the way in which bees in nature search for food, and the way in which optimization algorithms search for an optimum of (given) combinatorial optimization problems.

Since its appearance it had various successful applications. BCO has been proposed for the first time in [1–3] for solving the travelling salesman problem and was evolving through later applications. The BCO has also been applied to: the vehicle routing problem [4], the routing and wavelength assignment (RWA) in all-optical networks [5], the ride-matching problem [6], the traffic sensors locations problem on highways [7], and the static scheduling of independent tasks on homogeneous multiprocessor systems [8].

In this paper we develop a new version of the BCO algorithm to deal with the problem of locating p centers on a network of n vertices (p-center problem) with symmetric distance matrix. *Center problems* in location analysis are the problems related to the location of emergency-type facilities (centers) in the transportation networks. Ambulance, fire and police departments all over the world have numerous facilities (fire stations, police stations, ambulances depots). Proper location of these facilities, as well as adequate vehicle dispatching strategy has a crucial influence on a survival rate of citizens who are in emergency-type situations. For emergency services, the location of the facilities must be such as to minimize the furthest distance (time) that the vehicles will ever travel when responding to an emergency call. The importance of this issue is reflected in the large number of papers that consider the location of centers [9–19].

To approach the p-center problem, we propose an artificial system composed of a number of precisely defined agents (individuals, artificial bees). Multi-agent simulation is performed, i.e. we specify behavior rules of our agents and simulate the interaction between them (the way in which agents communicate with each other). Our model and a corresponding computer program determine the way our agents perform their activities. Various behavior rules lead us to different variants of BCO algorithms. Two variants of the BCO algorithm can be distinguished straightforward: constructive BCO and BCOi, the variant based on the improvement concept. As of the authors' knowledge, the BCOi was not used in the relevant BCO literature so far, and here it proves to be very efficient. Therefore, the development of BCOi represents major contribution of this work.

To test the efficiency of our implementation we performed a set of numerical experiments on well-known benchmark problems. We compared BCOi results with the state-of-the-art heuristic

☆ This work has been supported by Serbian Ministry of Science and Technological Development, Grant nos. 144007 and 144033.
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0305-0548/$ - see front matter
doi:10.1016/j.cor.2010.12.002

algorithms. Reported results show that the BCOi is competitive with other methods with respect to both comparison criteria: solution quality and running time. Moreover, BCOi was able to generate three (out of 40) new best known solutions within negligible CPU times.

The rest of this paper is organized as follows. Section 2 is devoted to the description of BCO technique, while the description of p-center problem and actual implementation of BCOi for solving it are given in Section 3. Section 4 contains experimental evaluations and Section 5 concludes the paper.

2. Bee colony optimization (BCO)

The bee colony optimization (BCO) meta-heuristic [1–3] belongs to the class of *nature-inspired algorithms*—the algorithms based on various biological and natural processes. We are particularly interested in *swarm behavior* based on the biological needs of individuals to stay together.

When staying together, individuals have a higher chances to defend themselves, find food and therefore to stay alive. Typical examples of swarm behavior are flocks of birds, parliament of owls, herds of land animals, fish schools and various social insects (bees, wasps, ants, termites) colonies. Swarm behavior is primarily characterized by autonomy, distributed functioning and self-organizing. The communication systems between individual insects contribute to the pattern called the "collective intelligence" of the social insect colonies.

The term "swarm intelligence" that denotes this "collective intelligence", has been introduced in [20] and it is the branch of Artificial Intelligence. Swarm intelligence is based on investigation of actions of individuals in different decentralized systems. These decentralized systems (multi agent systems) are composed of physical individuals (robots, for example) or "virtual" (artificial) ones that communicate among themselves, cooperate, collaborate, exchange information and knowledge and perform some tasks in their environment. Scientists use principles discovered in nature to build artificial systems. The development of artificial systems usually does not involve the entire imitation of natural systems, but explores them while searching for ideas and models.

As the examples of artificial multi agent systems we mention ant colony optimization (ACO) [21], particle swarm optimization (PSO) [22] and various algorithms inspired by bees' behavior. BCO was among the first of bees-inspired algorithms, it appeared at the very beginning of this century. Among other algorithms inspired by honeybees we point out the following. Yonezava and Kikuchi [23] analyzed collective intelligence based on bees' behavior. Sato and Hagiwara [24] proposed modified genetic algorithm named bee system. In essence, this algorithm belongs to the class of genetic algorithms. Abbas [25] developed *MBO* model that is based on the marriage process in honeybees. *BeeHive* [26–28], *artificial bee colony* (ABC) algorithm [29–31] and *bees algorithm* [32–34] as well as BCO, are based on foraging behavior of honeybees, but all of them use different concepts for algorithm development. An interesting survey of the bees' behavior inspired algorithms could be found in [35,36].

The BCO meta-heuristic [1–3] has been proposed quite recently by Lučić and Teodorović (they used the term "bee system" in their first paper). The BCO is inspired by foraging behavior of honeybees. The basic plan behind the BCO is to build the multi agent system (colony of artificial bees) able to efficiently solve hard combinatorial optimization problems. The artificial bee colony, as any other nature-inspired object behaves partially similar, and partially in a different way from bee colonies in nature. The brief introduction to biological background of BCO follows in Section 2.1 and then BCO algorithm is described in details in Section 2.2.

2.1. BCO biological background

In order to describe basic idea of the BCO algorithm we are interested in principles and habits used by bees in nature when collecting food. Bee colonies in nature usually consist of 50,000–100,000 individual bees that cooperate in everyday activities. Majority of them are involved in exploring the fields in the neighborhood of their hive looking for food. They collect and accumulate nectar for later use.

Typically, in the initial step, some scouts search the region. Completing the search, scout bees return to the hive and inform their hive-mates about the locations, quantity and quality of available food sources in the areas they have examined. If they discover nectar in the previously investigated locations, scout bees dance in the so-called "dance floor area" of the hive, in an attempt to "advertise" food locations and encourage (recruit) the remaining members of the colony to follow their lead. The information about the food quantity is presented using a ritual called a "waggle dance". If a bee decides to leave the hive and collect nectar, it will follow one of the dancing scout bees to the previously discovered patch of flowers. Upon arrival, the foraging bee takes a load of nectar and returns to the hive relinquishing the nectar to a food storer.

Several scenarios are then possible for a foraging bee: (1) it can abandon the food location and take a role of an uncommitted follower; (2) it can continue with the foraging behavior at the discovered nectar source, without advertising it to the others; (3) it can try to recruit its (uncommitted) hive-mates with the dance ritual before the return to the food location. The bee opts for one of the above alternatives. As several bees may be attempting to recruit their hive-mates at the dance floor area at the same time, it is unclear how above decisions are made, although it has been concluded that "the recruitment among bees is always a function of the quality of the food source" [37]. The described process continues repeatedly, while the bees from a hive accumulate nectar and explore new areas with potential food sources.

2.2. BCO algorithm

The basic idea of designing BCO is to build the multi agent system (colony of artificial bees) that will search for good solutions of various combinatorial optimization problems exploring the principles used by honey bees during nectar collection process. Artificial bee colony usually consists of small number of individuals (for example 5 or 10), but nevertheless, BCO principles are gathered from natural systems. Artificial bees investigate through the search space looking for the feasible solutions. In order to find better and better solutions, autonomous artificial bees collaborate and exchange information. Using collective knowledge and sharing information among themselves, artificial bees concentrate on more promising areas, and slowly abandon solutions from the less promising ones. Step by step, artificial bees collectively generate and/or improve their solutions. The BCO search is running in iterations until some predefined stopping criteria is satisfied.

Population of agents (artificial bees) consisting of B bees collaboratively searches for the optimal solution. Every artificial bee is responsible for one solution to the problem. There are two alternating phases (*forward pass* and *backward pass*) constituting single step in the BCO algorithm. In each forward pass, every artificial bee explores the search space. It applies a predefined number of moves, which construct and/or improve the solution, yielding a new solution. For example, let bees Bee 1, Bee 2, ..., Bee B participate in the decision-making process on n entities. At each forward pass bees are supposed to select one entity. Entity could be