

Portfolio adjusting optimization with added assets and transaction costs based on credibility measures

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ABSTRACT

In response to changeful financial markets and investor's capital, we discuss a portfolio adjusting problem with additional risk assets and a riskless asset based on credibility theory. We propose two credibilistic mean–variance portfolio adjusting models with general fuzzy returns, which take lending, borrowing, transaction cost, additional risk assets and capital into consideration in portfolio adjusting process. We present crisp forms of the models when the returns of risk assets are some deterministic fuzzy variables such as trapezoidal, triangular and interval types. We also employ a quadratic programming solution algorithm for obtaining optimal adjusting strategy. The comparisons of numeral results from different models illustrate the efficiency of the proposed models and the algorithm.

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1. Introduction

By assuming the lending and borrowing of money at a given riskless interest rate, Sharpe (1964) proposed the capital asset pricing model (CAPM). With the introduction of a riskless asset, an investor is able to put part of his or her money in riskless asset and the remainder in any of the risky portfolios contained in a feasible set. Sharpe et al. (1999) pointed out that adding these new opportunities expands the feasible set significantly and, more important, changes the location of a substantial part of an efficient set. The changes concern investors. Portfolio selection research, such as Best and Hlouskova (2000) and Zhang and Wang (2008), take the riskless asset into account and show a significant distinction between portfolio selection of only considering risk assets and portfolio selection of considering the existence of riskless asset.

Most of the existing portfolio selection models are based on probability theory and assume that the returns of assets are random variables. However, in real world, the probabilistic approaches only partly capture the reality. Although many events are characterized as fuzzy by probabilistic approaches, they are not random events (see Huang, 2010). Portfolio selection financial environment often subjects to fuzziness, and investment decision makers are subject to vagueness and ambiguity. It is more reasonable to assume that asset returns as fuzzy variables. Therefore, in some situations in which it is difficult to use probability theory,

investors can make use of fuzzy variables to reflect the vagueness and ambiguity of asset returns. In fact, more and more research extended probabilistic mean–variance models in Markowitz (1952, 1959) to fuzzy environment in different ways, such as Tarrazo (1997), Inuiguchi and Tanino (2000), Carlsson et al. (2002), León et al. (2002), Giove et al. (2006), Bilbao-Terol et al. (2006), Zhang et al. (2007, 2009), Huang (2007, 2010), Liu (2008), and Zhang et al. (2010). In fuzzy portfolio optimization, credibility theory is developed by Liu and Liu (2002) in which credibility measure is self-dual. Each fuzzy variable is associated with a credibility distribution in a similar way that each random variable is associated with a probability distribution. In this aspect, the credibility measure shares some properties like the probability measure. We use credibility measures to describe the fuzzy returns of risk assets in this paper.

The situations in financial markets are changeful, so an existing portfolio may not be efficient after a period of time. The changed data and capital imply that a change should be made in an investor's holdings. Therefore, it is necessary to develop adjusting problems for an existing portfolio in response to changed situation in financial markets and the investor's capital. In practical situations, there is a cost associated with buying or selling a risk asset. This is called the transaction cost. Many scholars, such as Patel and Subrahmanyam (1982), Arnott and Wanger (1990), Morton and Pliska (1995), Yoshimoto (1996) and Fang et al. (2006), studied portfolio selection problems with transaction cost and showed that the transaction cost has obviously a direct impact on investment performance. In the portfolio adjusting problem, the transaction cost is a V-shaped function of difference between a new portfolio and the existing portfolio, since the revision entails both purchasing and selling risk assets that both incur transaction costs.

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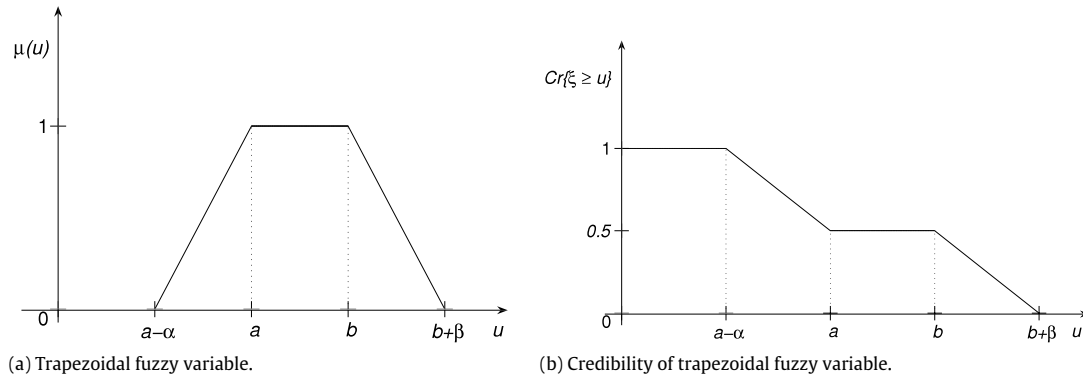


Fig. 2.1. Trapezoidal fuzzy variable and its credibility.

Moreover, in a real portfolio selection problem, investors do not only purchase or sell risk assets, but also lend or borrow a riskless asset (capital) at different interest rates.

The purpose of this paper is to discuss the portfolio adjusting problem based on credibility theory in response to changed data and capital. We use credibilistic expected value of fuzzy returns as the investment return and credibilistic variance as the risk. Different from Huang (2007) and Zhang et al. (2010), we discuss complex situations with additional risk assets and a riskless asset (capital) with different interest rates on lending and borrowing in portfolio adjusting process. Also, we consider the buying or selling transaction costs for the existing risk assets and the buying transaction costs for the additional risk assets.

The rest of the paper is organized as follows. In Section 2, we introduce the definitions of credibility expected return and credibilistic risk. Section 3 formulates two types of credibilistic portfolio adjusting models with or without borrowing when the returns of assets are general fuzzy variables. In Section 4, the proposed portfolio adjusting optimization models are converted to crisp forms for some deterministic fuzzy variable types. We also employ a quadratic programming solution algorithm for obtaining optimal strategy. In Section 5, a series of numeral examples are given to illustrate the effectiveness of the proposed models and algorithm. Finally, some concluding remarks are given in Section 6.

2. Credibilistic expected return and credibilistic risk of asset

In this paper we use fuzzy numbers to represent the uncertainty of the future return on assets. Let ξ denote the return of asset and be a fuzzy variable with membership function $\mu(\cdot)$. The credibilistic expected value of ξ is defined in Liu and Liu (2002) as

$$E_c[\xi] = \int_0^\infty Cr\{\xi \geq u\}du - \int_{-\infty}^0 Cr\{\xi \leq u\}du, \tag{1}$$

where

$$Cr\{\xi \leq u\} = \frac{1}{2} \left(\sup_{x \leq u} \mu(x) + 1 - \sup_{x > u} \mu(x) \right)$$

is called the credibility of a fuzzy event $\{\xi \leq u\}$. The value of credibility takes values in $[0, 1]$ and Cr is self-dual, i.e., $Cr\{\xi \leq u\} + Cr\{\xi > u\} = 1$.

Based on the concept of the credibility expected value, credibilistic variance of ξ is defined by

$$V_c[\xi] = E_c[(\xi - E_c[\xi])^2] = \int_0^\infty Cr\{(\xi - E_c[\xi])^2 \geq r\}dr. \tag{2}$$

For any real numbers λ_1 and λ_2 , the credibility expected value and credibilistic variance have the following properties

$$\begin{aligned} E_c[\lambda_1\xi + \lambda_2] &= \lambda_1 E_c[\xi] + \lambda_2 \quad \text{and} \\ V_c[\lambda_1\xi + \lambda_2] &= \lambda_1^2 V_c[\xi], \end{aligned} \tag{3}$$

respectively.

Especially, if ξ is a trapezoidal fuzzy number with tolerance interval $[a, b]$, left-width $\alpha > 0$ and right-width $\beta > 0$, then its membership function has the following form:

$$\mu(u) = \begin{cases} \frac{u - (a - \alpha)}{\alpha}, & \text{if } a - \alpha \leq u \leq a, \\ 1, & \text{if } u \in [a, b], \\ \frac{(b + \beta) - u}{\beta}, & \text{if } b \leq u \leq b + \beta, \\ 0, & \text{otherwise,} \end{cases}$$

and the graph of the membership function of trapezoidal fuzzy variable is shown in Fig. 2.1(a).

Thus, the credibility of event $\xi \geq u$ is as follows:

$$Cr\{\xi \geq u\} = \begin{cases} 1, & \text{if } u \leq a - \alpha, \\ \frac{\alpha - u + a}{2\alpha}, & \text{if } a - \alpha \leq u \leq a, \\ \frac{1}{2}, & \text{if } u \in [a, b], \\ \frac{\beta - u + b}{2\beta}, & \text{if } b \leq u \leq b + \beta, \\ 0, & \text{otherwise,} \end{cases}$$

and the graph for the credibility of event $\xi \geq u$ is illustrated in Fig. 2.1(b).

From (1) and (2), it follows that the credibilistic expected value and the credibilistic variance of a trapezoidal fuzzy number $\xi = (a, b, \alpha, \beta)$ are obtained as

$$E_c[\xi] = \frac{2a + 2b - \alpha + \beta}{4}, \tag{4}$$

and

$$\begin{aligned} V_c[\xi] &= \frac{4\theta^2 + 3\theta\tau + \tau^2 + 9\theta\gamma + 3\tau\gamma + 6\gamma^2}{48} \\ &\quad + \frac{[(\theta - \tau - 2\gamma)^+]^3}{384\theta}, \end{aligned} \tag{5}$$

respectively, where $\theta = \max\{\alpha, \beta\}$, $\tau = \min\{\alpha, \beta\}$, $\gamma = b - a$ and $(\theta - \tau - 2\gamma)^+ = \max\{\theta - \tau - 2\gamma, 0\}$.

In particular, if $\alpha \leq \beta \leq 2(b - a) + \alpha$, the credibilistic variance of ξ can be simplified as

$$V_c[\xi] = \frac{\beta^2 + \alpha^2 + 3[\beta + (b - a)][\beta + \alpha + 2(b - a)]}{48}. \tag{6}$$

In the following discussion, we use the credibilistic expected value as the return measure of risk asset and the credibilistic variance as the risk measure of risk asset.

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