A new method to cope with decision makers' uncertainty in the equipment selection process

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Abstract

One of the main problems faced while configuring or reconfiguring manufacturing systems is to rank alternative designs taking into account all the different aspects involved (both tangible and intangible). For this purpose the Analytic Hierarchy Process (AHP) is a well-known decision making support method that addresses this problem. A major drawback of AHP is that uncertainty in the judgments of the decision makers and the resulting impact on the ranking is not considered. In real situations, however, judgments based on perceived future scenarios are almost always uncertain. To solve this problem in this paper we present the first complete probabilistic extension to the AHP method. The new method provides the decision maker not only with information on the ranking of the alternatives but also the probability that the ranking remains stable even in presence of uncertainty in the judgements. We verified the validity of the new method in a real application developed for the Ferrari racing team.

Keywords:

Decision making, Flexible manufacturing system (FMS), Analytic hierarchy process (AHP)

1 INTRODUCTION

1.1 The Analytic Hierarchy Process (AHP)

Multi criteria decision making is a very complex problem in modern industry, as clearly stated by Krause and Golm [1] and by Elwany, Khairy, Abou-Ali and Harraz [2].

Analytic Hierarchy Process is a decision making support instrument, developed by T. L. Saaty in 1980 [3], useful to for managing multi criteria decisions. AHP has been widely used in industry in cases of investment selection, make or buy decisions, supplier selection, and configuration or reconfiguration of manufacturing systems.

The method consists in tackling the selection among alternatives, a complex multidimensional decisional problem, by using a hierarchy of comparisons based on simpler criteria. At each level the decision maker defines the weight of any alternative referred to a single criterion and then he evaluates the relative importance of the criteria in the context of the decision at hand. AHP allows the tackling of problems taking into account both its tangible and intangible aspects. AHP also allows the formalization of the decisional process, in order to better justify the decisions and later to find the causes of possible mistakes. Therefore AHP is also a company knowledge management tool and favours the professional growth of the people involved in its use.

At any level of the decisional tree pairwise comparisons are used to give the weights to the criteria within the hierarchy and to the alternatives at the bottom of it. Each judgement defines the relative importance of two items when compared referring to the higher level criterion in the hierarchy.

Therefore at each level in the hierarchy we can define a matrix of judgements. In such a matrix the condition that $a_{ij}=1/a_{ji}$ holds. Obviously the principal diagonal is filled with ones. Saaty showed that a pairwise comparison judgements' matrix can have just one eigenvalue and that the correspondent eigenvector, normalized to 1, can be a very good measure of the relative importance of the alternatives. Therefore we call the eigenvector the "*priority vector*".

1.2 Uncertainty in AHP

One of the main problems of the classical AHP methodology is that it does not take into account uncertainty in the judgments since the matrixes of judgements are deterministic. In real applications the decision makers are always subject to uncertainty while expressing their judgments and do not like to be forced to give deterministic answers. Moreover by eliminating uncertainty from the judgement it becomes impossible to evaluate its impact on the final decision's uncertainty. This limitation greatly reduces the applicability of the AHP methodology and also reduces the confidence of the users on the final results of the AHP techniques.

To overcome this limitation in the literature two different families of approaches for the problem are proposed: the fuzzy approach and the probabilistic approach. In the former, judgements are represented as fuzzy variables and methods are proposed to derive the priority vectors as fuzzy variables. In the latter, judgements can be represented as random variables. The goal is to find the distributions of the priority vectors and of the rankings of the variables.

While most of the papers appeared so far to concentrate on the fuzzy approach, we propose in this paper a probabilistic approach. The main advantage of the probabilistic approach is that operations on random variables are based on a unique definition while in the case the fuzzy sets different definitions of the fuzzy operations lead to different methods and different results which are difficult to compare.

A probabilistic approach adds to a standard AHP approach new parameters such as the probability each decisional alternative has to be the best one, which allows the decision makers to cope with uncertainty.

2 THE PROBABILISTIC AHP METHOD

2.1 Literature review

We refer to three fundamental articles regarding the AHP's probabilistic extensions. In the first article[4], Vargas tries to demonstrate that if the judgements in a matrix are gamma distributed variables then the eigenvector follows a Dirichlet, or multinomial Beta, distribution. The main merit of this paper is that it is the first to provide a probabilistic approach to the problem of uncertainty in AHP. However there are two problems with this approach. First the gamma is not reciprocal, therefore defining an element a_{ij} of the matrix as a gamma random variable, we have that $a_{ji}=1/a_{ij}$ is not distributed as a gamma. Therefore the order in which the decision maker gives his judgements modifies the probability distributions of the elements of the matrix, and this goes against one of the main principle of the AHP methodology. Secondly the elements of the principal diagonal are treated as random variables but by definition they must be equal to 1.

The second article we refer to [5], proposed by Escobar and Moreno-Jimenez, solves both problems, demonstrating that if a judgement follows a reciprocal distribution then also its reciprocal is a random variable that follows the same kind of distribution. Moreover also the priority vector's components follow a reciprocal distribution. In particular the authors demonstrate the validity of these conclusions for the lognormal distribution.

The limit of this approach is that it is impossible to evaluate analytically the distributions of the various random variables in particular the rankings and the global priority vectors within the tree. Therefore to obtain the required information it is necessary to use the MonteCarlo simulation.

The third article [6] we consider, the most important for our proposal, is a work published by Dennis in 1987. Referring to Wilks' work [7] Dennis demonstrates that, under some conditions involving the parameters, if every weight vector in the decisional tree is a Dirichlet distributed vector, then also the global priority vectors follow the same kind of distribution.

2.2 The new probabilistic AHP method

We propose to represent the pairwise comparison judgements with second kind Beta variables. With this approach we do not incur the problems encountered by Vargas since the functions are reciprocal and we can also exploit the Dennis method to obtain a complete probabilistic approach.

The variables used in the pairwise comparisons have the following probability distribution:

$$Be_2(x,\alpha,\beta)=\frac{x^{\alpha-1}}{Be(\alpha,\beta)\cdot(1+x)^{\alpha+\beta}} \quad E[Be_2(x,\alpha,\beta)]=\frac{\alpha}{\beta} \tag{2.1}$$

In the proposed approach we represent the components of a Saaty nxn matrix with second kind Beta variables and we define n parameters α_k so that each element a_{ij} of the matrix follows a distribution $Be_2(\alpha_i,\alpha_j)$. Under these conditions we can affirm that the components of the eigenvector are standard Gamma random variables, with parameters $\alpha_1,\dots,\alpha_n$. We can easily verify this statement in the case of perfect consistency of the matrix. Indeed a second kind Beta variable $Be_2(\alpha_i,\alpha_j)$ corresponds to the ratio between two standard Gamma variables with parameters α_i, α_j. Therefore we can write the judgments matrix as:

$$\begin{bmatrix} 1 & Be_2(\alpha_1,\alpha_2) & \cdots & Be_2(\alpha_1,\alpha_n) \\ Be_2(\alpha_2,\alpha_1) & 1 & \cdots & Be_2(\alpha_2,\alpha_n) \\ \vdots & \vdots & \ddots & \vdots \\ Be_2(\alpha_n,\alpha_1) & Be_2(\alpha_n,\alpha_2) & \cdots & 1 \end{bmatrix} = \tag{2.2}$$

$$= \begin{bmatrix} 1 & \Gamma(\alpha_1)/\Gamma(\alpha_2) & \cdots & \Gamma(\alpha_1)/\Gamma(\alpha_n) \\ \Gamma(\alpha_2)/\Gamma(\alpha_1) & 1 & \cdots & \Gamma(\alpha_2)/\Gamma(\alpha_n) \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma(\alpha_n)/\Gamma(\alpha_1) & \Gamma(\alpha_n)/\Gamma(\alpha_2) & \cdots & 1 \end{bmatrix}$$

Knowing that in case of perfect consistency every Gamma with the same parameter in 2.2 is exactly the same variable, it is obvious that the eigenvalue of the matrix is n and that the eigenvector is the following:

$$v \approx [\Gamma(\alpha_1) \quad \dots \quad \Gamma(\alpha_n)]^T \tag{2.3}$$

In the rest of the paper we will continue to represent the variables with their distributions. We will also use apexes to distinguish between variables with the same distribution.

Now we demonstrate the previous result in the case of non-perfect consistency. Using the well known eigenvector formula $A\cdot v=k\cdot v$ we can deduce n different expressions for the eigenvalue k:

$$k=1+\sum_{\substack{j=1\\j\neq i}}^{n}\frac{a_{ij}\cdot v_j}{v_i} \tag{2.4}$$

If the hypothesis that the eigenvectors' elements are gamma distributed is verified, then for each row of the system the eigenvalue is:

$$k_i \approx 1+\sum_{\substack{j=1\\j\neq i}}^{n}\left[Be_2(\alpha_i,\alpha_j)\Gamma(\alpha_j)/\Gamma(\alpha_i)\right]$$

Remembering that the product $Be_2(\alpha_i,\alpha_j)\Gamma(\alpha_j)$ follows a $\Gamma(\alpha_i)$ distribution, we deduce the following:

$$k_i \approx 1+\sum_{\substack{j=1\\j\neq i}}^{n}Be_2(\alpha_i,\alpha_i) \tag{2.6}$$

Now we show that two second kind Beta variables with respectively parameters (α_i,α_i) and (α_j,α_j) are equivalent:

$$\begin{gathered} Be_2(\alpha_i,\alpha_j)\cdot\Gamma(\alpha_j)=\Gamma'(\alpha_i) \\ \Downarrow \\ \frac{\Gamma(\alpha_i)}{\Gamma'(\alpha_j)}\cdot\Gamma(\alpha_j)=\Gamma'(\alpha_i) \\ \Downarrow \\ Be_2(\alpha_j,\alpha_j)=\frac{\Gamma(\alpha_j)}{\Gamma'(\alpha_j)}=\frac{\Gamma'(\alpha_i)}{\Gamma(\alpha_i)}=Be_2(\alpha_i,\alpha_i) \end{gathered} \tag{2.7}$$

In this way it is demonstrated that the representation of k as the sum of second type beta variables can be the same for every row of the matrix. Obviously this representation is valid only under some conditions. Indeed, because of the principles of the reciprocal matrices, the eigenvalue k cannot assume a value of less than n, and this condition cannot be verified if the variables are totally independent. The conditions to verify are in number of $n-1$ and involve the equivalence of the eigenvalue of the various rows of the system:

$$k_1=k_i \qquad \forall i=2,\dots,n \tag{2.8}$$

We show, as an example, the development of these conditions in the case of a 3x3 matrix:

$$\begin{cases} 1+Be_2(\alpha_1,\alpha_1)+Be_2'(\alpha_1,\alpha_1)=k \\ 1+Be_2(\alpha_1,\alpha_1)^{-1}+Be_2(\alpha_2,\alpha_2)=k \\ 1+Be_2'(\alpha_1,\alpha_1)^{-1}+Be_2(\alpha_2,\alpha_2)^{-1}=k \end{cases} \tag{2.9}$$

$$\begin{cases} Be_2(\alpha_1,\alpha_1)+Be_2'(\alpha_1,\alpha_1)=Be_2(\alpha_1,\alpha_1)^{-1}+Be_2(\alpha_2,\alpha_2) \\ Be_2(\alpha_1,\alpha_1)+Be_2'(\alpha_1,\alpha_1)=Be_2'(\alpha_1,\alpha_1)^{-1}+Be_2(\alpha_2,\alpha_2)^{-1} \end{cases} \tag{2.10}$$

$$\begin{cases} Be_2(\alpha_1,\alpha_1)=Be_2(\alpha_2,\alpha_2) \\ Be_2(\alpha_1,\alpha_1)=Be_2'(\alpha_1,\alpha_1)^{-1} \end{cases} \tag{2.11}$$

The eigenvalue is the following random variable:

$$k \approx 1+Be_2(\alpha_1,\alpha_1)+\frac{1}{Be_2(\alpha_1,\alpha_1)} \tag{2.12}$$

Finally since the eigenvector's elements are standard Gamma variables also in the case of non consistency the weights follow a Dirichlet distribution with parameters α_j for $j=1,\dots,n$ as already demonstrated by Vargas.