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Statistical properties of post-sample hedging effectiveness ☆

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Abstract

This paper examines the mean and the variance of post-sample hedging effectiveness. It is shown that, the hedging effectiveness measure adopted in the current literature is a biased estimator of the true hedging effectiveness. Moreover, it underestimates the true hedging effectiveness. Empirical results base upon twenty-four futures markets for the error correction hedge ratio, however, suggest the bias is negligible. On the other hand, in some markets, the variance of the hedging effectiveness is too large for the estimate to be reliable.

Keywords: Futures markets; Hedging effectiveness; Post-sample analysis

1. Introduction

A futures market serves as a financial instrument for risk reduction. To evaluate the usefulness of a futures market, one frequently turns to the hedging effectiveness measure proposed by Ederington (1979). Specifically, one constructs a hedge strategy such that the resulting hedged portfolio obtains the minimum variance of the return among all possible portfolios. Hedging effectiveness is measured by the percentage reduction of the minimum variance from the variance of the spot return. Different assumptions on the statistical behaviors of spot and futures prices lead to different minimum variance hedge strategies. Regardless, the true hedging effectiveness is derived from a true minimum variance hedge ratio.

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In empirical studies, hedging effectiveness is usually computed with the estimated hedge ratio replacing the true unknown hedge ratio. Lien (2006) demonstrates analytically that the method produces a downward biased estimator of the true hedging effectiveness. It therefore underestimates the usefulness of a futures contract. An unbiased estimator that corrects the bias is then proposed. This paper investigates empirically the size of the bias using weekly data from twenty-four financial and commodity futures markets.

Specifically, error-correction models are applied to generate estimated minimum variance hedge ratios and calculate the hedging effectiveness. It is found that the downward bias is very small. Consequently, bias correction seems unnecessary at least for direct hedging scenarios examined in the current paper. When examining the variance of hedging effectiveness, we find that, in hogs and cotton markets, the variance of the hedging effectiveness is too large for the estimate to be reliable.

The remainder of this paper is organized as follows. The next section discusses the true minimum variance hedge ratio and the true hedging effectiveness. We then describe the estimation methods of the hedge ratio and the hedging effectiveness. The downward bias of the estimated hedging effectiveness is characterized and the correction method is proposed. The subsequent section provides a detailed analysis for the error correction hedge ratio. The variance of the estimated hedging effectiveness is derived in the following section. An empirical study using weekly data from twenty-four futures markets is performed and analyzed. The final section concludes the paper.

2. Analytical hedging effectiveness

When we know the true data generation process (DGP), we can calculate the true minimum variance (MV) hedge ratio as the ratio of the conditional covariance between spot and futures prices over the conditional variance of the futures price:

$$h_t^* = \mathrm{Cov}(P_{t+1}, F_{t+1}|I_t)/\mathrm{Var}(F_{t+1}|I_t), \tag{1}$$

where P_{t+1} and F_{t+1} are, respectively, spot and futures prices at time $t+1$; I_t is the information available at time t. Frequently, spot and futures prices are found to contain a unit root. As a result, we have

$$h_t^* = \mathrm{Cov}(p_{t+1}, f_{t+1}|I_t)/\mathrm{Var}(f_{t+1}|I_t), \tag{2}$$

where $p_{t+1}=P_{t+1}-P_t$ and $f_{t+1}=F_{t+1}-F_t$. When conditional variances and the conditional covariance are stationary, the true MV hedge ratio is constant over time and we can replace h_t^* with h^*. The usefulness of a futures contract is calculated by the hedging effectiveness measure (Ederington, 1979), which is the percentage reduction in variance when the MV hedge ratio is applied. Let I_k be a $(k\times k)$ dimensional identity matrix and let e_k be a k-dimensional vector such that all elements equal to one. Define $M=I_k-(e_k e_k'/k)$. The hedging effectiveness of a futures contract is

$$H^* = 1-\frac{w'Mw}{p'Mp}, \tag{3}$$

where p is a k-dimensional vector consisting of k spot returns and w is a k-dimensional vector consisting of k hedged portfolio returns. A hedged portfolio return at time s is calculated as $p_s-h^*f_s$. As a result, $p'Mp$ and $w'Mw$ are sample variances of spot returns and hedged portfolio returns, respectively. In the following, we consider the statistical properties for the estimator of the Ederington measure. Nonetheless, it should be pointed out this measure has its own limitations (Lien, 2005a,b).