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Time-varying Hurst exponent for US stock markets☆

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ABSTRACT

In this work, the dynamical behavior of the US stock markets is characterized on the basis of the temporal variations of the Hurst exponent estimated with detrended fluctuation analysis (DFA) over moving windows for the historical Dow Jones (1928–2007) and the S&P-500 (1950–2007) daily indices. According to the results drawn: (i) the Hurst exponent displays an erratic dynamics with some episodes alternating low and high persistent behavior, (ii) the major breakthrough of the long-term trend of the scaling behavior occurred in 1972, at the end of the Bretton Woods system, when the Hurst exponent shifted form a positive to a negative long-term trend. Other effects, such as the 1987 crisis and the emergence of anti-correlated behavior in the recent two years, are also discussed.

1. Introduction

Exogenous and endogenous shocks can have a significant effect on the dynamics of financial markets. Endogenous shocks arise from the financial system itself and are commonly related to the behavior of investors. It has been argued that the 1987-market crash was partially induced by an overreaction of uninformed investors, resulting in panic selling and abrupt, dramatic price slips [1]. Exogenous effects, such as the 9/11-event, had a negative effect on the investor dynamics with a \$ 1.2 trillion lost in value for the week. Some news can have a positive effect, leading to a better functioning of the financial markets, such as annual reports on the economy expectations which can drive the financial indexes into historical high records, as has been occurred recently in China's stock exchange system. Although many of the exogenous and endogenous events usually produce a short-term significant effect on the financial markets, such effects are damped within relatively small time periods. For instance, before the 9/11-events the Dow Jones Index was about 11,000 points. In the events' week, the index fell into about 8200 point levels, and by the end of the year, the market recovered itself by attaining 10,200 point score. On the other hand, it seems that the 1987-market crash effects were reverted within the subsequent 4 months, and the markets evolved according to their internal forces by driving prices up and down only by supply and demand forces.

The examples commented above indicate that even though some exogenous events have had an important impact in the market dynamics, the effects were damped within a relatively small time period. This suggests that the market behaves like a dynamical system with fast and ample initial response to some exogenous shocks underlying quickly vanishing short-term destabilizing mechanisms followed by a damped response dominated by long-term stabilizing mechanisms. This paper

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addresses the problem of whether the stock markets had suffered significant shifts in their long-term behavior. This is done by testing on the variations over time of the return correlations for a large time period. In doing so, it is used a model-free statistical approach – detrended fluctuation analysis – that reduces the effects of non-stationary market trends and focuses on the intrinsic auto-correlation structure of market fluctuations over different time horizons.

The application of DFA for studying the stock market behavior was initiated by Liu et al. [3] by showing that the S&P-500 stock index exhibits weak long-range correlations. Grau-Carles [4] found little evidence of long-range correlations in returns for several stock markets, including the main US stock indices. Coronel-Brizio et al. [5] used 3-year windows over the period from 1978-2006 to show that, in the long-range, the US stock market operates close to the efficient market hypothesis (EMH). A recent study by Serletis et al. [6] confirmed that over long time scales, the US stock market indices are consistent with the EMH, although they do not exclude the possibility of market inefficiencies at shorter time scales. In principle, the knowledge of the stock market correlations allow some prediction degree that can be used for market characterization or investment strategies design. In this work, in the spirit Grecha and Mazur's approach [7], we test for time-varying degrees of long-range dependence for the major US stock indices using DFA. Specifically, significant temporal changes in the correlations of the Dow Jones (1928–2007) and the S&P-500 (1950–2007) daily indices are investigated by drawing the temporal evolution of the Hurst exponent on the basis of detrended fluctuation analysis (DFA) over a sliding window. Following the observation by Serletis et al. [6], we track the evolution of the Hurst exponent over short time-scales. Our results show that the Hurst exponent displays erratic behavior with some alternating episodes of low and high persistent behavior, with a major breakthrough in long-term trend of the Hurst occurring in 1972, coinciding with the end of the Bretton Woods system. Before 1972, the Hurst exponent long-term trend was positive indicating that the market efficiency (in the Fama's sense) were decreasing. After 1972, the long-term trend of the Hurst exponent became negative with approach towards efficient market behavior. The coincidence of the Hurst exponent change with the end of the Bretton Woods reference system in 1971–1972 suggests that this event had a major effect in the change of the markets behavior. The effect of other financial and social events, such as the 1987 market crash, are also discussed in the light of the Hurst exponent dynamics.

With respect to previous studies and results, our contribution can be summarized as follows:

- A rolling window DFA implementation showed that the most significant change in the stock market correlations are found over the period 1971–1972. Since this change coincides with the end of the Bretton Woods system, an impact of this on the US stock market is suggested.
- Recent US stock markets presented a shift towards anticorrelated behavior. In this paper, we argue that this reflects an change on stock investment strategies induced by incremented market uncertainties.

The paper is organized along the following lines. The data is described in Section 2. The methodology employed in the paper is commented in Section 3. The empirical results are described in Section 4. The results are put in perspective with previous studies in Section 5. The paper is closed in Section 6 with some concluding remarks.

2. Data

The data, which describe the historic behavior of Dow Jones and S&P-500 stock indices, were extracted from the public internet site finance.yahoo.com. The data contain opening, higher, lower and closing daily indices. In this work, as a representative daily index, the mean between opening and closing values was considered. The data, presented in Fig. 1(a, b), correspond to non-deflated indices displaying a Brownian-like behavior. The daily records are described in business time in the sense that the time units correspond to business days (b-day in the sequel). Returns were computed as $r_t = (I_t - I_{t-1})/I_{t-1}$, with I_t being the index at time t, and plotted in Figs. 1 and 2(b).

3. Methodology

The methodology employed to process the data is based on the assessment of correlation properties by applying the detrended fluctuation analysis (DFA) to the return time series. The DFA method, developed by Peng et al. [2], is a fractal scaling method commonly applied for detecting long-range correlations (*i.e.*, memory) in non-stationary sequences.

Briefly, the DFA method can be described as follows. For a given stochastic time series $y(i)$, $i = 1, \ldots, M$, with sampling period Δt, the standard DFA method consists in the following steps: (i) Compute the mean $\bar{y} = \frac{1}{M}\sum_{j=1}^{M} y(j)$, and the integrated time series $x(i) = \sum_{j=1}^{i} [y(j) - \bar{y}]$, $i = 1, \ldots, M$. (ii) Divide the integrated time series $x(i)$ into boxes of equal size m, and perform a box-wise sequence fit, denoted by $z_{lin}(i; m)$, to represent the local trend in each box, and iii) Compute the fluctuation sequence $w(i; m) = x(i) - z_{lin}(i; m)$, $i = 1, \ldots, M$, and the fluctuation function $F(m) = \left(\frac{1}{M}\sum_{j=1}^{M} |w(j; m)|^2\right)^{1/2}$. Repeat the above procedure for a broad range of box sizes m. When the signal follows a scaling law within a given scale domain $m \in D$, a power-law behavior for the fluctuation function $F(\tau)$, with $\tau = m\Delta t$, is observed: $F(\tau) \sim \tau^{\alpha}$, where α is called the scaling or Hurst exponent, a self-affinity parameter representing the long-range correlation properties of the signal. The lower and upper scale limits $m_{\min} \simeq 5$ and $m_{\max} \simeq M/5$ were recommended by Peng et al. [3]. For box sizes smaller than $m_{\min} \simeq 5$ the fluctuation function $F(\tau)$ carries out deterministic components induced by sampling effects.