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RESEARCH PAPER

Unbiased Estimation, Price Discovery, and Market Efficiency: Futures Prices and Spot Prices

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Abstract: In most cases, futures prices are not unbiased estimates of future spot prices. The price discovery function of futures markets should be defined as the lead-lag relationship between current futures prices and current spot prices, not the unbiased estimation of future spot prices. The pricing efficiency and information efficiency of futures markets are different. Three empirical models appropriate to investigate the relationship between futures prices and spot prices were discussed. As an application, these models were used to test the pricing efficiency, the lead-lag relationship, and the information efficiency of the S&P 500 index spot and futures markets from 21 September 1990 to 20 December 2007.

Key Words: futures markets; unbiased estimates; price discovery; market efficiency

Futures prices[1] are naturally highly related to spot prices[2] because futures are derivatives of spot assets. However, there are certain long-existing misunderstandings about the relationship between these two prices in academics and in financial industry. The purpose of this article is to clarify these misunderstandings and illustrate the real relationship between futures prices and spot prices.

1 Futures prices and spot prices: a review

There exist two types of researches on the relationship between futures prices and spot prices.

1.1 Current futures prices and the expected future spot prices[3]

Because futures prices represent the prices at which market participants agree to transact on a set date in the future, a conclusion seems natural that current futures prices F_t should be the prediction of future spot prices S_T in an efficient market, which is defined as the function of "price discovery" of futures. In futures markets literature, this argument is expressed as "futures prices are unbiased estimates of future spot prices", that is $F_t = E\left(S_T|I_t\right)$[4]. This view is broadly believed.

In academics, one example is the Pure Expectation Theory of the interest rate term structure proposed by Fisher in 1896. In modern financial language, this theory actually regards forward rates as unbiased estimates of future spot rates[5]. Another example is the well-known Unbiased Forward Exchange Rate Hypothesis (UFER), which argues that forward exchange rates are unbiased estimates of future spot exchange rates. Similar opinions exist extensively in many other futures markets. Accordingly, many researchers consider the failure of unbiased estimation as evidence of market inefficiency, such as Leuthold[2] , Martin & Garcia[3], Hokkio & Rush[4] and Bhattacharya & Singh[5], etc. In their articles, they all relate the issue of $F_t = E\left(S_T|I_t\right)$ to the efficiency of futures market.

The development of "theories" brings a mass of empirical studies. To our knowledge, at least hundreds of articles try to use different kinds of econometrics techniques to investigate the efficiency of currency futures markets, commodity futures markets, or other futures markets by testing $F_t = E\left(S_T|I_t\right)$. However, so far no convincing evidence is obtained so that $F_t = E\left(S_T|I_t\right)$ could not be either supported or rejected. For example, the Unbiased Forward Exchange Rate Hypothesis is regarded as a puzzle in finance (literatures [6] and [7].).

In industry, the opinion that futures prices are unbiased estimates of future spot prices is also deeply ingrained in peoples mind. One typical example is people keep using the fed funds futures rates for forecasting future spot fed-

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[1] It could be proved that when the risk-free interest rate is constant and the same for all maturities, the forward price for a contract with a certain delivery date in theory is the same as the futures price for a contract with that delivery date. When interest rates vary unpredictably (as they do in the real world), they are no longer the same in theory. However, the theoretical differences are in most circumstances sufficiently small to be ignored in reality. In this article, we assume that futures and forward contracts are just the same. Anything we discuss about futures will be the same for forwards.

[2] Hereafter, we use spot assets to denote the underlying assets corresponding to the futures contracts we discuss. Underlying assets in this article are all investment assets, not consumption assets.

[3] In this article, we refer to the markets average opinion about what the spot price of an asset will be at maturity of futures contracts discussed as the expected spot price of the asset.

[4] Here, T is the time until delivery date in a futures contract, t denotes the current time point, which is prior to the delivery date. I_t is the information set at time t and $E(\cdot|I_t)$ is the conditional expectation operator at time t.

[5] There exist three different versions of this theory. Cox, Ingersoll and Ross[1] give a detailed discussion.

eral funds rates, just as we can see in articles on *The Wall Street Journal*, *Financial Times*, or in the reports of some Fed Watchers (such as Altig[8] and Hamilton[9]) and some central banks[6].

1.2 Current futures prices and current spot prices

After 90's, cointegration tests and Granger causality tests were quite popular in financial researches, and empirical studies of the relationship between current futures prices F_t and current spot prices S_t became hot topics. Some researchers conduct cointegration analysis on F_t and S_t to investigate the pricing efficiency of futures markets, such as Brooks, et al.[12] and Crowder & Phengpis[13]. Some other researchers use Granger causality tests and vector error correction models (VECMs) to examine the lead-lag relationship and information flows between futures and spots markets, such as Wahab & Lashgari[14], Abhyankar[15] and Chiang & Fong[16]. Accordingly, some researchers define the lead-lag relationship and information flows between futures and spots markets as "price discovery". The market absorbing and reflecting new information more rapidly is said to have the function of price discovery.

As we can see, the above two types of researches are both concerning the efficiency of futures markets and the function of price discovery. Which one captures the real relationship between futures prices and spot prices ? Are they related to each other? Is the efficiency of futures market the same concept as the classical Efficient Market Hypothesis (EMH) in finance? How should we test the efficiency of futures markets? What on earth is the function of price discovery of futures markets?

The purpose of this article is to answer these questions. In brief, our conclusions include: 1) In most cases, current futures prices are not unbiased estimates of future spot prices. Futures do not have the function of "discovering" future spot prices. 2) However, current futures prices in one well-developed futures market would lead the change of current spot prices. It is more reasonable to define "price discovery" as the lead-lag relationship and information flows between two markets. 3) Different from the information efficiency of stock market defined in the classical EMH, the efficiency of futures market includes pricing efficiency and information efficiency. The former should be tested by investigating the relationship between F_t and S_t, whereas the latter should be tested by examining the residuals of futures log returns. 4) Anyhow, it is almost impossible to obtain reliable evidence about the market efficiency and the function of price discovery of futures markets by investigating the relationship between F_t and $E\left(S_T|I_t\right)$.

2 Futures prices and spot prices: A theoretical analysis

2.1 Pricing formulas of futures

Supposing that the no-arbitrage condition holds[7], futures prices satisfy

$$F_t = S_t e^{(r_t - q_t)(T-t)}, \tag{1}$$

where r_t is the continuously compounded risk-free interest rate, q_t is the average yield per annum on the spots during the life of the futures contracts with continuous compounding[8]. This equation is also called the cost of carry model. If Eq.(1) does not hold, arbitrageurs will execute arbitrage strategies until Eq.(1) holds again and the market reaches equilibrium.

However, if arbitrage is limited (for example, borrowings, lendings, and short sales are restricted, futures and spots are not good substitutions for each other.), the above no-arbitrage pricing formula will not hold any more. In such situations, theoretical futures prices like Eq.(1) are hard to obtain.

2.2 Current futures prices and current spot prices

From the above discussion, a natural conclusion is whether the relationship between F_t and S_t that satisfies Eq.(1) could be used to tell whether futures markets are efficient in pricing. Consider the natural logarithm on both sides of Eq.(1), we have

$$f_t = s_t + \left(r_t - q_t\right)\left(T - t\right), \tag{2}$$

where $f_t = \ln\left(F_t\right)$ and $s_t = \ln\left(S_t\right)$.

We know that log prices series are usually nonstationary. Thus, the step before we examine whether Eq.(2) holds is to test the stationarity of these three time series f_t, s_t, and $\left(r_t - q_t\right)\left(T - t\right)$. If they are all nonstationary, cointegration tests need to be conducted for $\{X_t\} = \left\{\left(f_t, s_t, \left(r_t - q_t\right)\left(T - t\right)\right)^T\right\}$. If these three series are cointegrated with the cointegration vector $(1, -1, -1)$, it could be thought that there exists long-term equilibrium among f_t, s_t, and $\left(r_t - q_t\right)\left(T - t\right)$ and the futures markets could be thought efficient in pricing. If $\left(r_t - q_t\right)\left(T - t\right)$ is stationary, we only need to perform cointegration tests on f_t and s_t to tell whether there is long-term equilibrium between these two prices. If f_t and s_t are cointegrated, the Granger causality test could be conducted on the basis of a VECM to examine the information flow and the lead-lag relationship between futures markets and spot markets. Or we could rearrange Eq.(2) and establish the following econometrics model

$$f_t - s_t = \alpha + \beta\left(r_t - q_t\right)\left(T - t\right) + \varepsilon_t. \tag{3}$$

We could then use regressions to investigate whether the relationship between the basis $f_t - s_t$ and the cost of carry satisfies the pricing efficiency (that is, to test whether $\alpha = 0, \beta = 1$ given ε_t is white noise.)

In one sentence, the pricing formula of futures prices explicitly shows that the investigation of the relationship between F_t and S_t could provide us evidence not only about the pricing efficiency of futures market but also about the information flow and the lead-lag relationship between markets. Consequently, it is quite reasonable to define "price discovery" as the information flow mechanism and the lead-lag relationship between futures markets and spot markets.

[6]For example, similar opinions could be found in European Central Bank Monthly Bulletin[10] and Federal Reserve Monetary Policy Report to Congress[11].

[7]In one perfect market, risk-free arbitrage is described as one trading strategy that requires no initial investment but guarantees a riskless profit higher than the riskless return.

[8]During the life of one futures contracts, a positive, negative, and zero average yield means $q_t > 0$, $q_t < 0$, and $q_t = 0$, respectively.