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Harvesting of dynamically complex consumer–resource systems: Insights from a threshold management policy

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ABSTRACT

Consumer–resource systems may present complex dynamical behavior due to multiplicity of steady states, initial conditions and/or critical parameter values. Threshold policies are one of the strategies employed to exploit these systems. It is shown that proper combinations of the threshold densities and harvesting effort intensities based on virtual equilibrium can yield desirable results from the management point of view despite the possible dynamical complexities in single as well as in multi-species population models.

1. Introduction

Consumer–resource dynamics models include, amongst others, predator–prey, host–parasitoid and herbivore–plant dynamics, which may present a myriad of dynamical behavior as a result of multiplicity of equilibrium states as well as different initial conditions and parameter values (Murdoch et al., 2003; Turchin, 2003). Therefore, the exploitation of such dynamically complex systems – which have straightforward applications – may be severely impaired by these factors. Along with constant harvest rate, fixed proportional harvest and fixed escapement level, threshold policy is an alternative strategy used to exploit the referred systems, and its application can be seen in areas such as fishery (Quinn and Deriso, 2000; Quinn et al., 1990; Collie and Spencer, 1993), terrestrial harvesting (Jozen et al., 2003), grazing (Noy-Meir, 1975), conflict uses of aquatic vegetation (Van Nes et al., 2002), control of nonnative predators (Sabo, 2005), to name a few.

In this work, given the variable structure (Utkin, 1978) of the threshold policy, it is shown that proper combinations of the threshold densities and harvesting effort intensities based on virtual equilibrium (Costa et al., 2000) can lead a variable of interest to a previously chosen level despite the possible dynamical complexities in single as well as in multi-species consumer–resource models.

In this case, the ultimate behavior of the system under this strategy consists of the so-called sliding motion-stabilization of the dynamics by means of a very rapid switching between the application and the interruption of the control action. This dynamical outcome, bearing in part on virtual equilibrium, could serve as an instrument for management applications, for instance, harvest maximization of renewable resources (Meza et al., 2005) and control of microbial populations in a chemostat (Costa and Meza, 2006).

The outline of the work is as follows. In Section 2 the mathematical definition of the policy is laid out. In Section 3 the proposed strategy is applied to single species models, such as harvesting of aquatic macrophytes with critical depensation. In Section 4 the Rosenzweig-MacArthur predator–prey model under constant harvest rates is analyzed when submitted to such policy. In Section 5 an intraguild predation system with chaotic behavior is analyzed under

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the proposed TP. Finally, a discussion of the results is presented.

2. Mathematical definition of the threshold policy

A threshold policy (hereafter called TP) (see Fig. 1), can be defined as the function $\phi(\tau)$, such that:

$$\phi(\tau) = \begin{cases} 1 & \text{if} \quad \tau > 0 \\ 0 & \text{if} \quad \tau < 0, \end{cases} \tag{1}$$

where τ is the threshold that should be chosen according to the problem to be solved.

Given a species N with a particular population dynamics, the proposed TP, ϕ, applied on that species will be given by:

$$\frac{dN}{dt} = f(N) - \phi(\tau)g(N) \tag{2}$$

where

$$\phi(\tau) = \begin{cases} 1 & \text{if} \quad \tau > 0 \\ 0 & \text{if} \quad \tau \le 0, \end{cases}$$

and

$$\tau = N - N_{th},$$

or equivalently,

$$\frac{dN}{dt} = \begin{cases} f(N) - g(N) & \text{if} \quad N > N_{th} \\ f(N) & \text{if} \quad N < N_{th} \end{cases}$$

where $f(N)$ is the species growth rate, $g(N)$ a density dependent function dictating any species removal rate, and N_{th} a threshold level. The control action is enacted whenever the N level is above N_{th}. Within this setting, this policy creates two systems with their own equilibrium points, separated by the threshold level (actually, two structures, and hence the name *variable structure system* (Utkin, 1977, 1978)). If the equilibrium points are located in their opposite regions, they are named *virtual equilibrium points*. Otherwise, they are called *real equilibrium points*. In case the locally stable equilibrium points are virtual, they will never be attained since the dynamics changes as soon as the trajectories cross the threshold N_{th}. From this setup a *sliding mode* (Utkin, 1992) along N_{th} may ensue, if in its vicinity the vector fields of both structures are directed toward each other (see Fig. 2; it is worth mentioning that the existence of virtual equilibria is not a necessary condition for the occurrence of a sliding regime, see Dercole et al. (2003)).

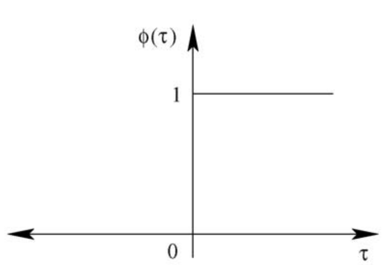

Fig. 1 – Threshold policy.

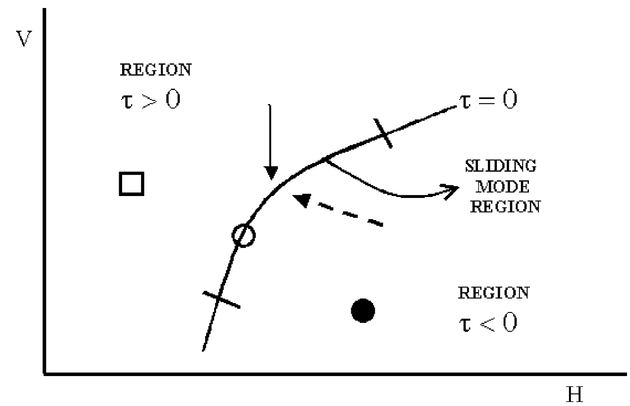

Fig. 2 – A diagrammatic figure of a sliding mode in a hypothetical phase plane $H \times V$. (□) Equilibrium point of $\tau < 0$; (●) equilibrium point of $\tau > 0$. These points are lying in opposite regions, hence they are virtual equilibrium points. (○) Equilibrium point of the sliding regime. Along the sliding mode region the vector fields of each structure (solid arrow, $\tau > 0$, dashed arrow, $\tau < 0$) are directed toward each other.

This dynamical behavior consists of rapidly alternating control activation and supression. As will be shown in the next examples, this strategy can stabilize complex dynamics by means of the creation of virtual equilibrium points (Costa et al., 2000; Meza et al., 2005).

3. Single species model

3.1. *Model of an aquatic vegetation system under harvesting with critical depensation*

Van Nes et al. (2002) analyzed the response of an aquatic plant population. V, subject to harvesting by means of the following model:

$$\frac{dV}{dt} = \left(\frac{2V^p}{V^p + H^p} - 1\right) rV\left(1 - \frac{V}{K}\right) - h_{max}\frac{V}{V + H_V}. \tag{3}$$

where K is the carrying capacity in which the net relative growth rate is zero and r the maximum relative growth rate at a very low biomass. It is taken into account the fact that vegetation can enhance its own growth conditions by clearing the water and by reducing erosion. The term $(2V^p/(V^p + H^p) - 1)$ describes this effect in a simple way. H^p is the critical vegetation biomass above which the presented term becomes positive and p determines the steepness of this raise. $h_{max}V/(V + H_V)$ is the Holling type II functional response. H_V is the vegetation biomass where the harvest is half the maximum harvesting rate h_{max}.

3.2. *Threshold policy applied to a model of harvesting an aquatic vegetation*

Putting (3) into the framework of variable structure systems to describe vegetation dynamics under such policy yields the follwing model: