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Demand modeling of stochastic product diffusion over the life cycle

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ABSTRACT

Planning during the product life cycle (PLC) poses a number of challenges for managers due to the pace of change and uncertainties in the marketplace. The ability to better understand, predict, and make decisions based on manifestations of demand forms a set of important operational problems that ultimately affect the profitability of enterprises. This paper models the stochastic diffusion of a product in the market as a geometric Brownian motion (GBM) process that has a time-varying drift rate. The model is calibrated such that model parameters are able to feature different product types and diffusion conditions. Imperfect information on the expected peak demand is treated as model uncertainty, and a Bayesian approach is employed to update knowledge on it. The demand model demonstrates robust performance over a wide range of conditions despite model uncertainty. It provides both qualitative and quantitative information for manufacturers and service providers to design strategies for stochastic PLC conditions as well as dynamic production planning.

1. Introduction

High-tech industries confront a unique set of challenges because their success depends on their capability to seize relatively short-lived opportunities and hedge risks by providing timely technology, appropriately skilled labor, and facilities. This desirable competence in quickly changing and uncertain environments is one common definition of agility. Information is critical to the positioning and implementation of agility strategies. Particularly, many high-tech products are characterized by product life cycles (PLCs), over which demand substantially changes in a partly stochastic manner (Pisano and Wheelwright, 1995). Knowledge of the PLC phenomenon will build a foundation for deploying agility strategies, thus allowing high-tech firms to effectively generate production profits under quickly changing, highly uncertain environments. Success during the current PLC also makes them more capable of introducing subsequent generations of products to the market.

The PLC phenomenon has been studied for several decades. Social scientists interpret it as a diffusion process, whereby a product penetrates consumer categories that have different behaviors and timing of adoption (e.g., Rogers, 1962; Katz et al., 1963). Diffusion theory has been found to be an appropriate scientific foundation for modeling the PLC process (Bass, 1969; Golder and Tellis, 2004). There have been two general modeling approaches, the Geometric Brownian Motion (GBM) model and the Bass (1969) model, which formulate the diffusion of products into the market from different perspectives.

GBM and Brownian Motion (BM) processes have become a popular family of stochastic models that formulate nonstationary demand to support dynamic capacity planning (see e.g., Luss, 1982; Van Mieghem, 2003; Wu et al., 2005). The applications of GBM and BM are broadly used to describe diffusion processes in economics, finance, management, physics, and biological systems (Karatzas and Shreve, 1998). GBM and BM modeling of nonstationary demand was introduced by Manne (1961). More recently, implementations of related models for dynamic capacity planning have been developed by Chou et al. (2007) and Qin and Nembhard (2010). A GBM process with a constant drift rate will not capture the demand for a product throughout its life cycle because the demand will at some point turn from growth to decline during its life. Bollen (1999) addressed the issue by modeling the demand during a product's life cycle as two concatenated GBM processes. Despite its usefulness, the Bollen model does not capture other potentially important features of product diffusion during the life cycle, which has led to the need for developing alternate models. This ongoing need for well calibrated models of PLC demand motivates the model development in this paper.

The Bass (1969) model is perhaps the most representative and widely used PLC model based on diffusion theory. The model shows that demand for, or the sales of, a product usually has a bell-shaped pattern thereby necessitating the adjustment of production scale. Other Bass-related models have improved upon this general approach. These include the SMPRT model recently

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developed by Chien et al. (2010). The family of Bass and Bass-like models are broadly used to support product design, production planning, and operations. Although a bell-shaped pattern is often observed for products with clear life cycles, the process of a new product spreading into the market may not be perfectly predicted by this pattern. Bass (2004) also noted that problems related to the stochastic PLC deserve additional attention.

In this paper we develop a new GBM-based model that explicitly describes the uncertainty in demand diffusion, and captures the major dynamic features of life cycles. The remainder of the paper is organized as follows. Section 2 outlines the related literature to summarize extant research. Section 3 introduces the proposed model followed in Section 4 by a summary of analytical PLC model behavior. In Section 5 we illustrate model implementation with a numerical example, which also helps to provide some practical characteristics of model behavior. Lastly, in Section 6 we discuss the broader set of findings, conclusions, and potential future research directions.

2. Literature

Many products progress through life cycles, wherein demand or concomitant sales undergo four stages including introduction, growth, maturity, and decline (Levitt, 1965). An early review of PLC research was performed by Rink and Swan (1979); however, social scientists and economists interpret the PLC phenomenon as a diffusion process, whereby a new product penetrates consumer categories that have different behaviors and timing of adoption (e.g., Rogers, 1962; Katz et al., 1963; Stoneman, 2002). The diffusion theory has formed the scientific foundation for studying the PLC phenomenon, and driven model development. It has also been broadly acknowledged by marketing researchers (e.g., Mahajan et al., 1990; Van den Bulte, 2000; Golder and Tellis, 2004). Recently, Chandrasekaran and Tellis (2007) performed a comprehensive review of marketing research on the PLC models, which lays a foundation for studying the PLC phenomenon in new product diffusion. The PLC concept is also validated from the perspectives of econometrics (e.g., Brockhoff, 1967), statistics (e.g., Polli and Cook, 1969), economics (e.g., Russel, 1980), and systems (e.g., Van den Bulte and Stremersch, 2004).

The PLC has been broadly modeled to support policy formulation, strategic planning, and operations (Day, 1981) and a seminal paper by Bass (1969) initiated a stream of PLC demand modeling research. For example, Bass (1995) further investigated reasons why sales have a bell-shaped pattern over the course of the product life cycle by examining general attributes of marketing such as price. Chien et al. (2010) built a multi-generation diffusion model, SMPRT, that performs well in forecasting semiconductor demand. Reviews of the Bass and related models were performed by Mahajan et al. (1990, 1995), Parker (1994), and Chandrasekaran and Tellis (2007), which thoroughly address various aspects of the models such as estimation techniques, forecasting abilities, applications, and extensions. Chandrasekaran and Tellis (2007) summarized limitations of Bass-related models and further suggested modeling improvements, wherein they concluded that model uncertainty (i.e., difficulty in providing accurate estimates of model parameters) limits predictive ability for very new products, and that Bayesian methods have been actively used to address this difficulty through updating parameter values when more information becomes available. They also emphasized the difficulty in identifying turning points within the PLC, which indicates that a continuous time model might be more adaptive than a four-stage model. Moreover, stochasticity in diffusion is an issue that must be addressed properly in order to make a PLC model reliable for decision support. For instance, Rosegger (1986) claimed that variations beyond the bell-shaped pattern are at least as important as the pattern itself, which could not be perfectly explained by deterministic diffusion models. Bass (2004) also stated that problems related to stochastic product life cycle deserve additional attention.

Another stream of research focuses on modeling the dynamics of stochastic diffusion as BM or GBM processes and using these models to support dynamic capacity planning (Luss, 1982; Van Mieghem, 2003; Wu et al., 2005). The seminal paper by Manne (1961) modeled the demand diffusion as a BM process, which has since been related to options pricing theory, in which the GBM is a commonly used underlying process (Broadie and Detemple, 2004). A series of work has used GBM as a demand model to support decisions in capital investment and capacity planning (e.g., Giglio, 1970; Pindyck, 1988; Bean et al., 1992; Dixit and Pindyck, 1994; Dixit, 1995; Eberly and Mieghem, 1997; Dangl, 1999; Benavides et al., 1999; Birge, 2000; Chou et al., 2007; Qin and Nembhard, 2010). Following Manne (1961), the literature dedicated to GBM demand modeling itself is sparse in relation to implementation of GBM demand models. Chou et al. (2007) empirically verified that GBM is appropriate for modeling semiconductor IC demand by representing demand uncertainty.

To model product demand across the entire life cycle, a GBM process with a constant growth rate of demand may lead to inaccurate conclusions since the rate can change direction. This is one motivation for having multiple phases of demand as was considered by Bollen (1999), wherein demand was modeled as a two-regime Geometric Brownian Motion (GBM) process: the life cycle begins with a growth regime and will stochastically switch to a decay regime. The expected growth rate is a positive constant for the growth regime, and for the decay regime it is a negative constant. While the model was shown to perform well under certain conditions, Golder and Tellis (2004) showed that growth rate varies significantly over four stages of the product life cycle. Moreover, the Bollen model does not explicitly encompass all important information of product diffusion, such as the expected market size and degree of technological innovation. Additionally, the stochastic switch in the Bollen model, when demand goes from growth to decay, may in general become less uncertain as more information becomes available.

3. The model

In this paper, we formulate the dynamics of demand during the PLC using a GBM model. Specifically, the model formulates the annual growth rate of demand over the product's life, which is common in PLC modeling (e.g., Pindyck, 1988; Bollen, 1999; Chandrasekaran and Tellis, 2007; Chou et al., 2007). Let D_t designate the demand at time t during the life cycle $[0,T]$, then dD_t/D_t is the relative change in demand within a small time interval $[t,t+dt]$. dD_t/D_t can vary with time because of the heterogenous diffusion rate during the PLC; it is also non-deterministic due to the stochasticity of product diffusion. A GBM process represents the dynamics of demand:

$$\frac{dD_t}{D_t} = \mu_t\, dt + \sigma\, dW_t, \tag{1}$$

where μ_t is the expected growth rate of demand at time t, σ is the standard deviation, and $(1/dt)(dD_t/D_t) - \mu_t$ represents the uncertainty in the growth rate of demand. W_t is the random walk process that models the randomness of diffusion (see pp. 176–177 in Neftci, 2000). Any change in W_t, dW_t, corresponding to time interval dt, satisfies $dW_t = \epsilon_t \sqrt{dt}$ ($\epsilon_t \sim N(0,1)$, and $E[\epsilon_t \epsilon_s] = 0$ for $t \neq s$). From the Central Limit Theorem, $W_{t+dt} - W_t \sim N(0,dt)$. The integral of the differential equation of demand dynamics