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Conditional volatility and correlations of weekly returns and the VaR analysis of 2008 stock market crash[☆]

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ABSTRACT

Modelling of conditional volatilities and correlations across asset returns is an integral part of portfolio decision making and risk management. Over the past three decades there has been a trend towards increased asset return correlations across markets, a trend which has been accentuated during the recent financial crisis. We shall examine the nature of asset return correlations using weekly returns on futures markets and investigate the extent to which multivariate volatility models proposed in the literature can be used to formally characterize and quantify market risk. In particular, we ask how adequate these models are for modelling market risk at times of financial crisis. In doing so we consider a multivariate *t* version of the Gaussian dynamic conditional correlation (DCC) model proposed by Engle (2002), and show that the *t*-DCC model passes the usual diagnostic tests based on probability integral transforms, but fails the value at risk (VaR) based diagnostics when applied to the post 2007 period that includes the recent financial crisis.

1. Introduction

Modelling of conditional volatilities and correlations across asset returns is an integral part of portfolio decision making and risk management. In risk management the value at risk (VaR) of a given portfolio can be computed using univariate volatility models, but a multivariate model is needed for portfolio decisions. Even in risk management the use of a multivariate model would be desirable when a number of alternative portfolios of the same universe of m assets are under consideration. By using the same multivariate volatility model marginal contributions of different assets towards the overall portfolio risk can be computed in a consistent manner. Multivariate volatility models are also needed for determination of hedge ratios and leverage factors.

The literature on multivariate volatility modelling is large and expanding. Bauwens et al. (2006) provide a recent review. A general class of such models is the multivariate generalized autoregressive conditional heteroskedastic (MGARCH) specification (Engle and Kroner (1995)). However, the number of unknown parameters of the unrestricted MGARCH model rises exponentially with m and its estimation will not be possible even for a modest number of assets. The diagonal-VEC version of the MGARCH model is more parsimonious, but still contains too many parameters in most applications. To deal with the curse of dimensionality the dynamic conditional correlations (DCC) model is proposed by Engle (2002) which generalizes an earlier specification by Bollerslev (1990) by allowing for time variations in the correlation matrix. This is achieved parsimoniously by separating the specification of the conditional volatilities from that of the conditional correlations. The latter are then modelled in terms of a small number of unknown parameters, which avoid the curse of the dimensionality. With Gaussian standardized innovations Engle (2002) shows that the log-likelihood function of the DCC model can be maximized using a two-step procedure. In the first step, m univariate GARCH models are estimated separately. In the second step using standardized residuals, computed from the estimated volatilities from the first stage, the parameters of the conditional correlations are then estimated. The two-step procedure can then be iterated if desired for full maximum likelihood estimation.

DCC is an attractive estimation procedure which is reasonably flexible in modelling individual volatilities and can be applied to portfolios with a large number of assets. However, in most applications in finance the Gaussian assumption that underlies the two-step procedure is likely to be violated. To capture the fat-tailed nature of

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the distribution of asset returns, it is more appropriate if the DCC model is combined with a multivariate t distribution, particularly for risk analysis where the tail properties of return distributions are of primary concern. But Engle's two-step procedure will no longer be applicable to such a t-DCC specification and a simultaneous approach to the estimation of the parameters of the model, including the degree-of-freedom parameter of the multivariate t distribution, would be needed. This paper develops such an estimation procedure and proposes the use of devolatized returns computed as returns standardized by realized volatilities rather than by GARCH type volatility estimates. Devolatized returns are likely to be approximately Gaussian although the same cannot be said about the standardized returns (Andersen et al. (2001a,b)).

The t-DCC estimation procedure is applied to a portfolio composed of 6 currencies, four 10 year government bonds, and seven equity index futures over the period May 27, 1994 to October 30, 2009; split into an estimation sample (1994 to 2007) and an evaluation sample (2008 to 2009). To avoid the non-synchronization of daily returns across markets in different time zones we estimate the volatility models using weekly rather than daily returns.

Main features of the empirical results are as follows:

- The estimation results strongly reject the normal-DCC model in favour of a t-DCC specification.
- The t-DCC specification passes the non-parametric Kolmogorov–Smirnov tests, but fails the VaR test due to the extreme events in September and October of 2008.
- Important changes to asset return volatilities have taken place which are shared across assets and markets.
- The 2008 financial crisis resulted in the reversal of the trend volatilities from its low levels during 2003–2007 to unprecedented heights in 2008.
- Asset return correlations have been rising historically. The recent crisis has accentuated this trend rather than leading to it.
- The rise in asset return correlations seems to be more reflective of underlying trends — globalization and integration of financial markets, and cannot be attributed to the recent financial crisis. More research on this topic is clearly needed.

The plan of the paper is as follows. Section 2 introduces the t-DCC model and discusses the devolatized returns and the rational behind their construction. Section 3 considers recursive relations for real time analysis. The maximum likelihood estimation of the t-DCC model is set out in Section 4, followed by a review of diagnostic tests in Section 5. The empirical application to weekly returns is discussed in Sections 6 and 7. The evolution of asset return volatilities and correlations is discussed in Section 8, followed by some concluding remarks in Section 9.

2. Modelling conditional correlation matrix of asset returns

Let $\mathbf{r}_t$ be an $m\times 1$ vector of asset returns at close day t assumed to have a conditional multivariate t distribution with means, μ_{t-1}, and the non-singular variance–covariance matrix Σ_{t-1}, and $v_{t-1}>2$ degrees of freedom. Here we are not concerned with how mean returns are predicted and take μ_{t-1} as given.[1] For specification of Σ_{t-1} we follow Bollerslev (1990) and Engle (2002) to consider the decomposition

$$\Sigma_{t-1} = \mathbf{D}_{t-1}\mathbf{R}_{t-1}\mathbf{D}_{t-1}, \tag{1}$$

where

$$\mathbf{D}_{t-1} = \begin{pmatrix} \sigma_{1,t-1} & & & \\ & \sigma_{2,t-1} & 0 & \\ & 0 & \ddots & \\ & & & \sigma_{m,t-1} \end{pmatrix},$$

$$\mathbf{R}_{t-1} = \begin{pmatrix} 1 & \rho_{12,t-1} & \rho_{13,t-1} & \cdots & \rho_{1m,t-1} \\ \rho_{21,t-1} & 1 & \rho_{23,t-1} & \cdots & \rho_{2m,t-1} \\ \vdots & & \ddots & & \vdots \\ \vdots & & & & \rho_{m-1,m,t-1} \\ \rho_{m1,t-1} & \cdots & \cdots & \rho_{m,m-1,t-1} & 1 \end{pmatrix},$$

$\mathbf{R}_{t-1} = (\rho_{ij,t-1}) = (\rho_{ji,t-1})$ is the symmetric $m\times m$ correlation matrix, and $\mathbf{D}_{t-1}$ is the $m\times m$ diagonal matrix with $\sigma_{i,t-1}, i=1,2,\ldots,m$ denoting the conditional volatility of the i-th asset return. More specifically

$$\sigma^2_{i,t-1} = V(r_{it}\,|\,\Omega_{t-1}),$$

and $\rho_{ij,t-1}$ are conditional pair-wise return correlations defined by

$$\rho_{ij,t-1} = \frac{Cov\left(r_{it}, r_{jt}\,|\,\Omega_{t-1}\right)}{\sigma_{i,t-1}\sigma_{j,t-1}},$$

where Ω_{t-1} is the information set available at close of day $t-1$. Clearly, $\rho_{ij,t-1}=1$, for $i=j$.

Bollerslev (1990) considers Eq. (1) with a constant correlation matrix $\mathbf{R}_{t-1}=\mathbf{R}$. Engle (2002) allows for $\mathbf{R}_{t-1}$ to be time-varying and proposes a class of multivariate GARCH models labeled as dynamic conditional correlation (DCC) models. An alternative approach would be to use the conditionally heteroskedastic factor model discussed, for example, in Sentana (2000) where the vector of unobserved common factors are assumed to be conditionally heteroskedastic. Parsimony is achieved by assuming that the number of the common factors is much less than the number of assets under consideration.

The decomposition of Σ_{t-1} in Eq. (1) allows separate specification of the conditional volatilities and conditional cross-asset returns correlations. For example, one can utilize the GARCH (1,1) model for $\sigma^2_{i,t-1}$, namely

$$V(r_{it}\,|\,\Omega_{t-1}) = \sigma^2_{i,t-1} = \bar{\sigma}_i^2(1-\lambda_{1i}-\lambda_{2i}) + \lambda_{1i}\sigma^2_{i,t-2} + \lambda_{2i}r^2_{i,t-1}, \tag{2}$$

where $\bar{\sigma}_i^2$ is the unconditional variance of the i-th asset return. Under the restriction $\lambda_{1i}+\lambda_{2i}=1$, the unconditional variance does not exist and we have the integrated GARCH (IGARCH) model used extensively in the professional financial community, which is mathematically equivalent to the “exponential smoother” applied to the r^2_{it}'s[2]

$$\sigma^2_{i,t-1}(\lambda_i) = (1-\lambda_i)\sum_{s=1}^{\infty}\lambda_i^{s-1}r^2_{i,t-s} \qquad 0<\lambda_i<1, \tag{3}$$

or written recursively

$$\sigma^2_{i,t-1}(\lambda_i) = \lambda_i\sigma^2_{i,t-2} + (1-\lambda_i)r^2_{i,t-1}. \tag{4}$$

[1] Although the estimation of μ_{t-1} and Σ_{t-1} are inter-related, in practice mean returns are predicted by least squares techniques (such as recursive estimation or recursive modelling) which do not take account of the conditional volatility. This might involve some loss in the efficiency of estimating μ_{t-1}, but considerably simplifies the estimation of the return distribution needed in portfolio decisions and risk management.

[2] See, for example, Litterman and Winkelmann (1998).