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Stock market dynamics

M.S. Baptista *, I.L. Caldas

Instituto de Física, Universidade de São Paulo, C.P. 66318, CEP 05315-970 São Paulo, S.P., Brazil

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Abstract

We elucidate on several empirical statistical observations of stock market returns. Moreover, we find that these properties are recurrent and are also present in invariant measures of low-dimensional dynamical systems. Thus, we propose that the returns are modeled by the first Poincaré return time of a low-dimensional chaotic trajectory. This modeling, which captures the recurrent properties of the return fluctuations, is able to predict well the evolution of the observed statistical quantities. In addition, it explains the reason for which stocks present simultaneously dynamical properties and high uncertainties. In our analysis, we use data from the S&P 500 index and the Brazilian stock Telebrás.

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1. Introduction

It is clear that the return distributions of stock market indexes does not have a Gaussian shape as proposed in Ref. [1], mainly due to the pronounced tail of these distributions. In Ref. [2] it is shown that the distribution of the fluctuations in cotton price is a stable Lévy distribution. However, due to the fact that the stable Lévy distribution has infinite variance, it does not fit well the decay of the distribution tail of the return indexes. The fact is that the asymptotic behavior of the return distribution shows faster decay than the one predicted by a Lévy distribution. Recently, a truncated unstable Lévy distribution, a Lévy distribution in the central part followed by an approximately exponential truncation, was proposed to describe the distribution of the return [3–5].

* Corresponding author. Tel.: +55-11-3091-7070; fax: +55-11-3091-6749.
E-mail address: murilo@if.usp.br (M.S. Baptista).

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Although a truncated Lévy distribution fits a return distribution, its tails follow a power-law asymptotic behavior, characterized by an exponent $\alpha \approx 3$, well outside the condition required for the Levy distribution stability ($0 < \alpha < 2$), as reported in Ref. [6].

Therefore, besides the relevant progress achieved in the statistical description of these fluctuations, a complete description of the return distributions has not been given in the previous works.

In recent works we showed that the return of the S&P 500 index has a Poisson-like distribution [7,8]. In agreement with this findings, in Ref. [9] the distribution of the increments of the British Pound/U.S.$ time series was found to decay as an exponential, a process whose distribution is a Poisson. Moreover, the observed Poisson-like distribution is equivalent to a Poisson-like distribution of the first Poincaré return time of a low-dimensional deterministic system. The first Poincaré return time measures the time a chaotic trajectory takes to return to a given reference interval in phase space.

One of the purposes of this work is to show that the distribution of the return of a stock index is the same for the recurrent time, i.e., the time the return takes to return to a specified interval of values. Furthermore, the other statistical properties of these two fluctuations can be simulated by measures of the first Poincaré return time of a low-dimensional trajectory. This equivalence, between the fluctuations of the return and a measure typical of chaotic dynamical systems, suggests that the stock market is dynamically recurrent, that is, there is a dynamical process ruling out the stock oscillations.

The use of dynamical tools, as the first Poincaré return time, can explain many empirical observations, described in the next section, for the return of a stock index [6,9,10]. In particular, the reason for the preservation, for long time scales, of the return distribution functional form. The model also describes very well how the average time intervals in which high return values occur (rare events) is related with the width of the return distribution (proportional to what is called volatility).

With the proposed procedure we explain properties and scales of the distributions for the return and the recurrent time of the S&P index and the Brazilian stock Telebrás.

This paper is organized as follows. In Section 2, we describe the many empirical observation for the index S&P 500 and the Brazilian stock Telebrás. In Sections 3–5, we describe that these observations can be well reproduced by using the first Poincaré return time, a dynamical variable, to simulate the returns. In Section 6, we present the conclusions of this work.

2. Data analysis

This paper shows and explain several empirical statistical manifestations observed in the stock market. These manifestations are: (i) The return distribution is a Poisson-like distribution; (ii) the return distribution functional form is preserved for long time periods, and therefore, it has a slow convergence to the Gaussian behavior; (iii) the asymptotic behavior of the return distribution follows a power law; (iv) the amplitude variation of the return with respect to time follows a typical scaling power law;