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Physica A 344 (2004) 284–288
PHYSICA A
www.elsevier.com/locate/physa

Investment strategy due to the minimization of portfolio noise level by observations of coarse-grained entropy

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Received 12 December 2003
Available online 20 July 2004

Abstract

Using a recently developed method of noise level estimation that makes use of properties of the coarse-grained entropy, we have analyzed the noise level for the Dow Jones index and a few stocks from the New York Stock Exchange. We have found that the noise level ranges from 40% to 80% of the signal variance. The condition of a minimal noise level has been applied to construct optimal portfolios from selected shares. We show that the implementation of a corresponding threshold investment strategy leads to positive returns for historical data.

PACS: 05.45.Tp; 89.65.Gh

Keywords: Noise level estimation; Stock market data; Time series; Portfolio diversification

1. Introduction

Although it is a common belief that the stock market behaviour is driven by stochastic processes [1–3], it is difficult to separate stochastic and deterministic components of market dynamics. In fact, the deterministic fraction follows usually

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doi:10.1016/j.physa.2004.06.133

from nonlinear effects and can possess a non-periodic or even chaotic characteristic [4,5]. The aim of this paper is to study the level of determinism in time series coming from stock market. We will show that our noise level analysis can be useful for portfolio optimization.

We employ here a method of noise-level estimation that has been described in details in Ref. [6]. The method is quite universal and it is valid even for high noise levels. It makes use of the functional dependence of coarse-grained correlation entropy $K_2(\varepsilon)$ [7] on the threshold parameter ε. Since the function $K_2(\varepsilon)$ depends in a characteristic way on the noise standard deviation σ, thus one can estimate the noise level σ by observing the dependence $K_2(\varepsilon)$. The validity of our method has been verified by applying it for the noise level estimation in several chaotic models [7] and for the Chua electronic circuit contaminated by noise. The method distinguishes a noise appearing due to the presence of a stochastic process from a non-periodic *deterministic* behaviour (including the deterministic chaos). Analytic calculations justifying our method have been developed for the Gaussian noise added to the observed deterministic variable. It has been also checked in numerical experiments that the method works properly for a uniform noise distribution and at least for some models with dynamical noise corresponding to the Langevine equation [6].

2. Calculations of noise level in stock market data

We define the noise level as the ratio of standard deviation of noise σ to the standard deviation of data σ_{data},

$$NTS = \frac{\sigma}{\sigma_{data}} . \tag{1}$$

Using this definition and our noise level estimation method [6], we have analyzed the noise level in data recorded at the New York Stock Exchange (NYSE). Let us consider logarithmic daily returns for the Dow Jones Industrial Average (DJIA):

$$x_i = \ln\left(\frac{P_i}{P_{i-1}}\right) . \tag{2}$$

Fig. 1 presents the plot of the noise level NTS for a corresponding time series x_i where values of NTS have been calculated as a function of a trading day. The noise level has been determined in windows of the length 3000 days and is pointed in the middle of every window. As one can see, the level of noise ranges from 60% to 90%, which makes any point to point forecasting impossible. We should mention that since the relative noise variance is NTS^2, thus in our case the noise variance is 40–80% of the data variance. It follows that there are time periods when the percent of an unknown deterministic part approaches the level 60% of the signal.

Similar estimations of the noise level have been performed for selected stocks of the NYSE. Results for the mean values of corresponding NTS parameters are presented in the Table 1. As one can expect, the noise level of a single stock is much larger than that for the DJIA. This is because deterministic parts of different stock