Research in Economics (2000) **54**, 403–416
doi:10.1006/reec.2000.0233
Available online at http://www.idealibrary.com on IDEAL®

Bayesian model comparison by Markov chain simulation: Illustration using stock market data

EFTHYMIOS G. TSIONAS†

Council of Economic Advisers, Ministry of National Economy, Athens, Greece

(*Received 21 May 1999, accepted 24 May 2000*)

Summary

The paper illustrates the computation of marginal likelihoods and Bayes factors when Markov Chain Monte Carlo has been used to produce draws from a model's posterior distribution. The method is based on Raftery (1996) and does not require that Gibbs sampling is used or conditional posterior distributions are available in closed form. Models used include a normal finite mixture, a GARCH and a Student *t*-model as alternative models for the Standard and Poor's stock returns.

J.E.L. Classification: C11, C15
Keywords: Markov chain Monte Carlo, Bayes factors, multivariate kernel estimation, GARCH models, normal discrete mixture, Student *t*-distribution, stock returns.

1. Introduction

The use of Markov Chain Monte Carlo (MCMC) techniques has revolutionized the field of applied Bayesian econometrics. Many models which were thought to be intractable prove to be amenable to methods of numerical integration organized around Markov Chain simulation, especially the Gibbs sampler and the Metropolis-Hastings algorithm. See Tanner and Wong (1987), Gelfand and Smith (1990), and Tierney (1994).

Although considerable progress has been made in the field of posterior simulation with arbitrary likelihoods after the introduction

† Address for correspondence: E.G. Tsionas, Council of Economic Advisers, Ministry of National Economy, 5 Nikis Street, Syntagma Square, Suite 607, Athens 10180, Greece. Tel: (301) 333-2558, Fax: (301) 333-2527, E-mail: soe3@compulink.gr

1090–9443/00/040403 + 14 $35.00/0

of Markov Chain Monte Carlo methods (Tanner & Wong, 1987; Tierney, 1994), the problem of computing marginal likelihoods and Bayes factors has proved extremely difficult. One possibility is to use Laplace integration (Tierney, Kass & Kadane, 1989) to approximate the required integrals. However, this does not take into account that a Markov Chain Monte Carlo sample from the posterior may be available. Kass and Wasserman (1992) suggested to use this sample and a quadratic regression of the log-posterior around the prior mean to estimate the mode and Hessian of the normal approximation. Other approaches estimate the integrals by drawing from the prior and averaging the likelihood. It is well known, however, that this approach is prone to numerical problems (McCulloch & Rossi, 1992).

Other approaches suggested by Gelfand and Dey (1994) and Newton and Raftery (1994) suffer from problems associated mainly with numerical instabilities, or require that all models be specified in advance (for example the approach by Carlin & Chib, 1993). More recently, Chib (1995) proposed an approach that seems very useful, but it is also associated with problems.

Chib (1995) noticed that the marginal likelihood can be estimated by evaluating the likelihood multiplied by the prior and dividing by an estimate of the posterior ordinate, both computed at a specific parameter value. Chib's approach does assume however that (1) Gibbs sampling is used, and (2) the integrating constants of the conditional distributions are known. These assumptions clearly exclude models where Gibbs sampling cannot be used because conditional sampling is impossible at low cost (e.g. ARCH or GARCH and stochastic volatility models, models with stable Paretian errors as in Tsionas (1999), for example) or models where integrating constants of conditional distributions are not available. In addition, the approach requires repeated Gibbs sampling for each conditioning argument, which is likely to be an overhead in most empirical applications. At any rate, the method can be applied *only when Gibbs sampling is feasible*, which is clearly restrictive.

Finally, Verdinelli and Wasserman (1995) proposed an interesting extension of the Savage-Dickey ratio which is, however, useful only when one wishes to test a sharp null hypothesis, without obvious extensions to the model comparison case. Like Chib's (1995) method it requires an additional simulation. The present paper presents a method that requires *no additional simulations at all* but only an evaluation of the posterior kernel at the posterior mean and a kernel density estimate at the same point. This method is due to Raftery (1996), was independently developed by Tsionas (1995), and applied to some complicated dynamic latent variable models by Wei (1996). The method is quite feasible computationally. Of