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Long term spread option valuation and hedging

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ABSTRACT

This paper investigates the valuation and hedging of spread options on two commodity prices which in the long run are in dynamic equilibrium (i.e., cointegrated). The spread exhibits properties different from its two underlying commodity prices and should therefore be modelled directly. This approach offers significant advantages relative to the traditional two price methods since the correlation between two asset returns is notoriously hard to model. In this paper, we propose a two factor model for the spot spread and develop pricing and hedging formulae for options on spot and futures spreads. Two examples of spreads in energy markets – the *crack spread* between heating oil and WTI crude oil and the *location spread* between Brent blend and WTI crude oil – are analyzed to illustrate the results.

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1. Introduction

Commodity spreads are important for both investors and manufacturers. For example, the price spread between heating oil and crude oil (*crack* spread) represents the value of production (including profit) for a refinery firm. If an oil refinery in Singapore can deliver its oil both to the US and the UK, then it possesses a real option of diversion which directly relates to the spread of WTI and Brent crude oil prices. There are four commonly used spreads: spreads between prices of the same commodity at two different locations (*location* spreads) or times (*calendar* spreads), between the prices of inputs and outputs (*production* spreads) or between the prices of different grades of the same commodity (*quality* spreads).[1]

A *spread option* is an option written on the difference (*spread*) of two underlying asset prices S_1 and S_2, respectively. We consider *European* options with *payoff* the greater or lesser of $S_2(T)$–$S_1(T)$–K and 0 at maturity T for *strike price K* and focus on spreads in the commodity (especially energy) markets (for both spot and futures). In pricing spread options it is natural to model the spread by modelling each asset price separately. Margrabe (1978) was the first to treat spread options and gave an analytical solution for strike price zero (the *exchange option*). Closed form valuation of a spread option is not available if the two underlying prices follow geometric Brownian motions (see Eydeland and Geman, 1998). Hence various numerical techniques have been proposed to price spread options, such as for example the Dempster and Hong (2000) fast Fourier transformation approach. Carmona and Durrleman (2003) offer a good review of spread option pricing.

Many researchers have modelled the spread using two underlying commodity spot prices (the two price method) in the unique *risk neutral measure* as[2]

$$\begin{aligned}
\mathrm{d}\mathbf{S}_1 &= (r-\delta_1)S_1\,\mathrm{d}t + \sigma_{1,1}S_1\,\mathrm{d}\mathbf{W}_{1,1},\\
\mathrm{d}\delta_1 &= k_1(\theta_1-\delta_1)\,\mathrm{d}t + \sigma_{1,2}\,\mathrm{d}\mathbf{W}_{1,2},\\
\mathrm{d}\mathbf{S}_2 &= (r-\delta_2)S_2\,\mathrm{d}t + \sigma_{2,1}S_2\,\mathrm{d}\mathbf{W}_{2,1},\\
\mathrm{d}\delta_2 &= k_2(\theta_2-\delta_2)\,\mathrm{d}t + \sigma_{2,2}\,\mathrm{d}\mathbf{W}_{2,2},
\end{aligned} \tag{1}$$

where $\mathbf{S}_1$ and $\mathbf{S}_2$ are the spot prices of the commodities and δ_1 and δ_2 are their convenience yields, and $\mathbf{W}_{1,1}$, $\mathbf{W}_{1,2}$, $\mathbf{W}_{2,1}$ and $\mathbf{W}_{2,2}$ are four

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[1] For more details on these concepts see Geman (2005a).

[2] Boldface is used throughout to denote random entities – here conditional on $\mathbf{S}_1$ and $\mathbf{S}_2$ having realized values S_1 and S_2 at time t which is suppressed for simplicity of notation.

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correlated Wiener processes. This is the classical Gibson and Schwartz (1990) model for each commodity price in a complete market.[3] The *return correlation* $\rho_{13} := E[d\mathbf{W}_{1,1} d\mathbf{W}_{2,1}]/dt$ plays a substantial role in valuing a spread option; trading a spread option is equivalent to trading the correlation between the two asset returns. However, Kirk (1995), Mbanefo (1997) and Alexander (1999) have suggested that return correlation is very volatile in energy markets. Thus assuming a *constant* correlation in (1) is inappropriate.

But there is another longer term relationship between two asset prices, termed *cointegration*, which has been little studied by asset pricing researchers. If a cointegration relationship exists between two asset prices the spread should be modelled *directly* over the long term horizon. Soronow and Morgan (2002) proposed a one factor mean reverting process to model the location spread directly, but do not explain under what conditions this is valid nor derive any results.[4] See also Geman (2005a) where diffusion models for various types of spread option are discussed.

In this paper, we use two factors to model the spot spread process and fit the futures spread term structure. Our main contributions are threefold. First, we give the first statement of the economic rationale for mean reversion of the spread process and support it statistically using standard cointegration tests on data. Second, the paper contains the first test of mean-reversion of *latent spot spreads* in both the risk neutral and market measures. Third, we give the first latent multi-factor model of the spread term structure which is calibrated using standard state-space techniques, i.e. Kalman filtering.

The paper is organized as follows. Section 2 gives a brief review of price cointegration together with the principal statistical tests for cointegration and the mean reversion of spreads. Section 3 proposes the two factor model for the underlying spot spread process and shows how to calibrate it. Section 4 presents option pricing and hedging formulae for options on spot and futures spreads. Sections 5 and 6 provide two examples in energy markets which illustrate the theoretical work and Section 7 concludes.

2. Cointegrated prices and mean reversion of the spread

A *spread* process is determined by the dynamic relationship between two underlying asset prices and the *correlation* of the corresponding *returns* time series is commonly understood and widely used. *Cointegration* is a method for treating the long run dynamic equilibrium relationships between two asset *prices* generated by market forces and behavioural rules. Engle and Granger (1987) formalized the idea of integrated variables sharing an equilibrium relation which turns out to be either stationary or to have a lower degree of integration than the original series. They used the term cointegration to signify co-movements among trending variables which could be exploited to test for the existence of equilibrium relationships within the framework of fully dynamic markets.

In general, the return correlation is important for short term price relationships and the price cointegration for their long run counterparts. If two asset prices are cointegrated (1) is only useful for *short term* valuation even when the correlation between their returns is known exactly. Since we wish to model long term spread we shall investigate the cointegration (long term equilibrium) relationship between asset prices. First we briefly explain the economic reasons why such a long run equilibrium exists between prices of the same commodity at two different locations, prices of inputs and outputs and prices of different grades of the same commodity.[5]

The *law of one price* (or *purchasing power parity*) implies that cointegration exists for prices of the same commodity at different locations. Due to market frictions (trading costs, shipping costs, etc.) the same good may have different prices but the mispricing cannot go beyond a threshold without allowing market arbitrages (Samuelson, 1964). Input (raw material) and output (product) prices should also be cointegrated because they directly determine supply and demand for manufacturing firms. There also exists an equilibrium involving a threshold between the prices of a commodity of different grades since they are substitutes for each other. Thus the spread between two spot *commodity* prices reflects the profits of producing (production spread), shipping (location spread) or switching (quality spread). If such long-term equilibria hold for these three pairs of prices, cointegration relationships should be detected in the empirical data.

In empirical analysis economists usually use Eqs. (2) and (3) to describe the cointegration relationship:

$$\mathbf{S}_{1t} = c_t + d\mathbf{S}_{2t} + \varepsilon_t, \tag{2}$$

$$\varepsilon_t - \varepsilon_{t-1} = \omega\varepsilon_{t-1} + \mathbf{u}_t, \tag{3}$$

where $\mathbf{S}_1$ and $\mathbf{S}_2$ are the two asset prices and $\mathbf{u}$ is a Gaussian disturbance. Engle and Granger (1987) demonstrate that the error term ε_t in (2) must be *mean reverting* (3) if cointegration exists. Thus the *Engle–Granger two step* test for cointegration directly tests whether ω is a significantly negative number using an augmented Dicky and Fuller (1979) test. Note that (2) can be seen as the dynamic equilibrium of an economic system. When the trending prices $\mathbf{S}_1$ and $\mathbf{S}_2$ deviate from the long run equilibrium relationship they will revert back to it in the future.

For both location and quality spreads $\mathbf{S}_1$ and $\mathbf{S}_2$ should ideally follow the *same* trend, i.e. d should be equal to 1.[6] Since gasoline and heating oil are cointegrated substitutes, the d value could be 1 for both the heating oil/crude oil spread and the heating oil/gasoline spread (Girma and Paulson, 1999). For our spreads of interest – production and location – d is treated here as 1.

Letting $\mathbf{x}_t$ denote the spread between two cointegrated spot prices $\mathbf{S}_1$ and $\mathbf{S}_2$ it follows from (2) and (3) in this case that

$$\mathbf{x}_t - x_{t-1} = c_t - c_{t-1} - \omega(c_{t-1} - x_{t-1}) + \mathbf{u}_t, \tag{4}$$

i.e., the spread of the two underlying assets is *mean reverting*. No matter what the nature of the underlying $\mathbf{S}_1$ and $\mathbf{S}_2$ processes,[7] the spread between them can behave quite differently from their individual behaviour. This suggests modelling the spread *directly* over a long run horizon because the cointegration relationship has a substantial influence in the long run. Such an approach

[3] We adopt this model for three reasons. (1) It fits futures contract prices much better than the one factor mean-reverting log price model as shown by Schwartz (1997). (2) In examining the historical commodity prices used here, we show that WTI and Brent crude oil and heating oil prices are *not* mean-reverting. This has been found by many others, e.g. Girma and Paulson (1999) and Geman (2005b). (We also show that the spread *is* mean-reverting.) Since the Gibson–Schwartz model has a GBM backbone, we believe it matches historical commodity prices better. (3) Schwartz (1997) shows that futures volatility in the one factor model will decay to close to zero after ten years, but using the Enron dataset he shows that the volatility for market futures with maturities longer than 2 years fluctuates around 12%. However the two factor Gibson–Schwartz model can match the volatility term structure quite well.

[4] We are grateful to an anonymous referee for this reference.

[5] Calendar spreads can be modelled using the models for individual commodities such as the models proposed by Schwartz (1997) and Schwartz and Smith (2000). In this paper two different commodities are considered.

[6] However for production spreads such as the *spark spread* (the spread between the electricity price and the gas price) d may not be exactly 1. Usually 3/4 of a gas contract is equivalent to 1 electricity contract so that investors trade a 1 electricity/3/4 gas spread which represents the profit of electricity plants (Carmona and Durrleman, 2003).

[7] Especially for commodities where many issues have to be considered, such as jumps, seasonality, etc. Hence no commonly acceptable model exists for all commodities.

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