Analysis

The extraction of natural resources: The role of thermodynamic efficiency

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ABSTRACT

The modelling of production in microeconomics has been the subject of heated debate. The controversial issues include the substitutability between production inputs, the role of time and the economic consequences of irreversibility in the production process. A case in point is the use of Cobb–Douglas type production functions, which completely ignore the physical process underlying the production of a good. We examine these issues in the context of the production of a basic commodity (such as copper or aluminium). We model the extraction and the refinement of a valuable substance which is mixed with waste material, in a way which is fully consistent with the physical constraints of the process. The resulting analytical description of production unambiguously reveals that perfect substitutability between production inputs fails if a corrected thermodynamic approach is used. We analyze the equilibrium pricing of a commodity extracted in an irreversible way. We force consumers to purchase goods using energy as the means of payment and force the firm to account in terms of energy. The resulting market provides the firm with a form of reversibility of its use of energy. Under an energy numeraire, energy resources will naturally be used in a more parsimonious way.

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1. Introduction

The central role of energy as an input in the production process of economically valuable goods is difficult to dispute. Yet in economics a consensus on how it should be modelled has not yet emerged. Intensive energy use creates environmental damage through the release of residual heat. As discussed in the seminal contribution of Georgescu-Roegen (1971), the second law of thermodynamics, which governs energy utilization and degradation, is a key source of negative externalities. The production literature has only dedicated a limited amount of attention to the concept of waste as an unavoidable joint product of any production process (Ayres and Kneese, 1969; Ethridge, 1973; Kummel, 1989, 1991). Traditional economic analysis of production generally avoids thermodynamic considerations. Typical production models require substitutability between all inputs, none of which, including energy, has a special role. On these premises, economics literature dealing with the use of energy has largely focused on the possibilities of substituting it as a factor of production in the presence of energy price shocks or energy shortages.

Factors of production (as well as consumption goods) are inter-changeable if they provide the same functionality. However, the complete substitutability between natural resources (including energy), labor and capital leads to paradoxical consequences. Daly (1997)

observes that, if labor and natural resources are substitutes and not complements then it would be possible to make a cake with "[…]only the cook and his kitchen. We do not need flour, eggs, sugar, etc., nor electricity or natural gas, nor even firewood. If we want a bigger cake, the cook simply stirs faster in a bigger bowl and cooks the empty bowl in a bigger oven that somehow heats itself […]". A dazzling example of this paradox can be found in the standard representation of funds and flows in the Cobb–Douglas production function model, that is:

$$Q = K^{\alpha_1} L^{\alpha_2} R^{\alpha_3}, \tag{1.1}$$

where Q is the output of the process per unit of time, K represents the stock of capital, L the labor supply and R the flow of natural resources. From expression (1.1) it is evident that we can obtain a fixed amount of output Q_0 and even if $R \to 0$, it is sufficient to choose an amount of K such that:

$$K = \left(\frac{Q_0}{L^{\alpha_2} R^{\alpha_3}} \right)^{\frac{1}{\alpha_1}} \to +\infty. \tag{1.2}$$

Although it is well understood that the Cobb–Douglas production function is only an abstract concept and not an actual description of a production process, this very concept has a pervasive influence on economic modelling. Substitutability plays a critical role in Neoclassical general equilibrium construction. It underlies the view that there are no real limits to economic growth as, in the extreme, "the world can, in effect, get along without natural resources" (Solow, 1974). In point of

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fact, Neoclassical general equilibrium also implies the substitutability of every good as a numeraire and means of payment. Still, this is a useful framework that yields insight on the value and optimal exploitation in time of non-renewable resources (Hotelling, 1931).

It may sensibly be argued, as Ayres and Miller (1980) put it, that there are definite and well-known limits on physical performance in almost every field, which derive from the unique role of some specific factor of production. Energy is a case in point. See Cleveland and Ruth (1997) for a survey of the literature covering this alternative perspective. However, such analyses are often of a qualitative nature. The case for (the lack of) substitutability can often be argued either way, due to the vagueness of the approach.

As a contribution to this debate, we provide a robust theoretical foundation for the lack of substitutability of the energy input in the extraction and refinement of a commodity. Specifically, we propose an analytic model for a mining operator who refines a mineral from its natural concentration to a strike concentration that defines the product.[1] The production process model is consistent with actual physical constraints and follows the thermodynamic theory of an irreversible separation process. [2]

All real world transformations involving energy are, in fact, irreversible. A thermodynamic transformation or cycle is said to be reversible if it is carried out by making infinitesimal changes to a state variable, which allow the system to be at rest throughout the entire process (Fermi, 1956). Such a transformation is impossible because it would require an infinite amount of time. All production processes are thus irreversible in nature, because they have to be carried out in a finite time-period in order to bring production to the market. They therefore involve a strictly positive increase in entropy $\Delta S > 0$. An increase in entropy means a reduction in "useful" energy, that is the part of energy that can be converted into work by an engine. Thus, an entropy increase can be interpreted as waste or resource degradation. If the production function does not accommodate the real thermodynamic process that leads to the final output, the impact of the producer's choices on resource depletion and the waste released into the environment will not be evident.

Despite attempts to minimize its use as an input, a positive energy amount is always necessary to extract the natural resource. This minimal requirement provides an answer to all issues raised by recent and past literature about substitutability between production process inputs. Although limited substitution of energy is possible, total substitution is impossible because there is a physical energy threshold, for a given quantity of raw material input, below which no production exists. This emphasises the importance of a physics-based approach to production modelling as a correct methodological way of resolving the substitutability issue.

Except for a few papers, microeconomic analysis completely avoids a detailed consideration of the physical constraints that are the essence of every production process.[3] Krysiak and Krysiak (2003) show that general equilibrium theory is consistent with the mass and energy conservation laws. Krysiak (2006) analyzes the consequences of the second law of thermodynamics on economic equilibrium in a general framework.[4] A production function derived from finite time

thermodynamic constraints can be found in Roma (2000, 2006). The production process we analyze here from a physical point of view differs from the thermal production process in Roma (2000, 2006) in that the final product obtained will be of homogeneous quality and unused energy can be stored.

An important precedent for our commodity extraction problem is found in Ruth (1995). This paper contains explicitly thermodynamic constraints on production, in the form of lower bounds on the inputs to be fed into a Cobb–Douglas production function. The lower energy bound is calculated for a reversible separation process. As mentioned above, this approach does not reflect the reality of production, which involves only irreversible processes.

We then turn to an analysis of the rational use of energy from an economics point of view. Berry et al. (1978) highlight the difference between the concepts of thermodynamic optimality of the use of energy in production and overall economic (cost) efficiency. They assume that the substitution of energy by other inputs along a production isoquant amounts to an increase in thermodynamic efficiency. As thermodynamic efficiency cannot be increased beyond the reversible limit, full substitutability of energy is prevented, i.e. it is impossible to move beyond a certain point on the isoquant in the energy dimension. The economic notion of Pareto optimality for resource allocation (where no individual can be better off without making at least one other worse off) will not in general amount to the optimal use of available energy from a thermodynamic point of view.

We advance the analysis proposed in Berry et al. (1978). We can analytically derive the production isoquant under the assumption of maximum finite time thermodynamic efficiency (a possibility dismissed by Berry et al. (1978) as optimistic, see p.133). This is still compatible with some (quite limited) substitutability between energy and raw material.

We analyse the problem of the optimal scale of production and the consequent exploitation of natural resources, including energy, under the finite time thermodynamic foundation adopted. Even if the most efficient thermodynamic technology is used, higher production in the same amount of time implies greater deviation from reversible efficiency and higher energy waste. Hence it is the scale and speed of production that ultimately determine thermodynamic efficiency.

We incorporate the production model in a simple general equilibrium framework in which we analyze inter-temporal production decisions. We derive the equilibrium in a two-period economy in which our good is produced and consumed. In a Neoclassical equilibrium, efficiency in the use of available energy will not, on its own, drive economic decisions. Under a Neoclassical model, the amount of thermodynamic waste will be irrelevant and natural resources will be fully exploited. On the other hand, the negative externalities associated with the degradation of energy, if taken into account, would lead to a thermodynamic efficiency criterion in the use of this scarce resource. However, thermodynamic efficiency would have to be imposed on the decision makers (by way, for example, of a "green tax"). We find, similarly to Roma (2006), that a competitive economic equilibrium will be twisted towards higher thermodynamic efficiency if energy is forced to be the numeraire and means of payment. This creates a market in which the energy price of production is established. When energy is the only scarce input, the producer will compare the energy cost of production with the energy value of the firm's sales, determined by supply and demand. The lack of substitutability between input factors and the decreasing return to scale feature of irreversible technology will prevent the complete use of available energy in production. This will in turn decrease energy degradation and thermodynamic waste. The neoclassical solution, where a change in numeraire and means of payment would not alter the resource allocation, is finally obtained if the production process is carried out over an infinite time, i.e. if it is reversible. Reversibility is equivalent to constant returns to scale technology.

This paper is organized as follows: Section 2 is dedicated to a description of the physical process underlying the production of the commodity. We derive the reversible limit of the production technology

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[1] A number of papers deal with the decisions concerning the exploitation of physical resources and the extraction/production of commodities (Brennan and Schwartz, 1985; Coratzar et al., 1998; Hartwick, 1978; Stiglitz, 1976) but none describe the thermodynamics of extraction.

[2] Separation processes are at the heart of many production processes and include distillation, evaporation, drying, deep freezing, centrifugation, membrane separation etc.

[3] Sav (1984) uses a micro-engineering production function derived from physical laws to model the exploitation of solar energy for domestic water heaters. Substitution elasticities between nonrenewable fuel inputs (oil or natural gas) and capital-intensive solar-produced heat are investigated.

[4] On the debate about the impact of the entropy law on economic equilibrium see also Young (1991) and Daly (1992), who lead to two completely opposite conclusions. Many authors have investigated Nicholas Georgescu-Roegen's paradigm of ecological economics, again obtaining conflicting conclusions, such as those reported by Khalil (1990), subsequently criticized by Lozada (1991) and finally re-stated by Khalil (1991).