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Multifractal behavior of the Korean stock-market index KOSPI

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Abstract

We investigate multifractality in the Korean stock-market index KOSPI. The generalized qth order height–height correlation function shows multiscaling properties. There are two scaling regimes with a crossover time around $t_c = 40$ min. We consider the original data sets and the modified data sets obtained by removing the daily jumps, which occur due to the difference between the closing index and the opening index. To clarify the origin of the multifractality, we also smooth the data through convolution with a Gaussian function. After convolution we observe that the multifractality disappears in the short-time scaling regime $t < t_c$, but remains in the long-time scaling regime $t > t_c$, regardless of whether or not the daily jumps are removed. We suggest that multifractality in the short-time scaling regime is caused by the local fluctuations of the stock index. But the multifractality in the long-time scaling regime appears to be due to the intrinsic trading properties, such as herding behavior, information outside the market, the long memory of the volatility, and the nonlinear dynamics of the stock market.

Keywords: Econophysics; Multiscaling; Multifractal; Stock market

1. Introduction

In recent years, concepts and techniques from statistical physics have been widely applied to economics [1–19], and the complex behaviors of economic systems have been found to be very similar to those of complex systems customarily studied in statistical physics. Stock-market indexes around the world have been accurately recorded for many years and therefore represent a rich source of data for quantitative analysis, and the statistical behaviors of stock markets have been studied by various methods, such as distribution functions [10–12,19], correlation functions [12–14], multifractal analysis [19–21], and network analysis of the market structure [15,16].

Multiscaling properties have been reported for many economic time series [22–31]. Multifractality has been observed in stock markets [20,29,32–34], the price of crude oil [25], the price of commodities [34], and foreign

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exchange rates [24,35]. In daily stock indexes and foreign exchange rates, the generalized Hurst exponent H_q decreases monotonically with q [24,29,32–35] (see formal definitions of H_q and q in Eqs. (1) and (2) below). In the US NASDAQ index, two scaling regimes have been reported: one quasi-Brownian and the other multifractal [33]. Two scaling domains have also been reported in the fluctuations of the price of crude oil: H_q increases with q in the short-time domain, but decreases with q in the long-time domain [25]. The existence of multiple scaling regimes seems to depend on the resolution of the data sets in the economic time series.

Although many economic time series display multifractality (in the theory of surface scaling analysis referred to as *multiaffinity*), the origins of multifractality in the stock market are not well understood. It has been suggested that herding behavior and nonlinear complex dynamics of the stock market induce multiscaling [10]. However, it is very difficult to quantify herding behavior and complex dynamics in the stock market. Buendía et al. observed multiaffinity in a frustrated spring-network model simulating the surface structure of cross-linked polymer gels [36]. Removing vertical discontinuities from the rough surface by convolution with a Gaussian, they observed that the multiaffine surface changed into a self-affine one [37]. They concluded that vertical discontinuities can be one cause of multiaffinity. Mitchell conversely introduced artificial vertical discontinuities into a self-affine surface and observed that the surface became multiaffine [38].

In the present paper we investigate multifractality in the Korean stock-market index (Korean Composite Stock Price Index (KOSPI)). We observe that local fluctuations, including jumps, are responsible for multifractality in the short-time scaling regime. However, multifractality in the long-time scaling regime is not removed by smoothing of the time series.

2. Method and results

We consider a set of data recorded every minute of trading from March 30, 1992, through November 30, 1999. We count the time during trading hours and remove closing hours, weekends, and holidays from the data. Denoting the stock-market index as $x(t)$, the generalized qth order height–height correlation function (GHCF) $F_q(t)$ is defined by

$$F_q(t) = \langle |x(t' + t) - x(t')|^q \rangle^{1/q}, \tag{1}$$

where the angular brackets denote a time average over the time series. The GHCF $F_q(t)$ characterizes the correlation properties of the time series $x(t)$, and for a multiaffine series a power-law behavior like

$$F_q(t) \sim t^{H_q} \tag{2}$$

is expected, where H_q is the generalized qth order Hurst exponent [39]. If H_q is independent of q, the time series is monofractal. If H_q depends on q, the time series is multifractal. Multifractality is a distinctive property of the stock-market index observed.

In Fig. 1(a) we present the GHCF as a function of the time interval t. We observe clear multifractal behavior in the time series of the stock index. There are two different scaling regimes separated by a crossover time of about $t_c = 40$ min. The slopes of the log–log plot depend on q in each scaling regime. As shown in Fig. 1(b), $H_q \sim 1/q$ for large q in both scaling regimes, while it saturates for small q. The $1/q$ behavior is consistent with numerical observations in Refs. [37,38] and analytical results for functions that include discontinuities in Ref. [38]. These behaviors are different from the multifractality in the price of crude oil [25]. Alvarez-Ramirez et al. reported the existence of two scaling regimes for crude-oil prices [25], but H_q *increased* with q in the long-time scaling regime. Kim et al. did not observe a short-time scaling regime for the daily Yen–Dollar exchange rate [22].

2.1. Removal of daily jumps

We analyzed the KOSPI time series to find the origin of multifractality in the Korean stock market. Inspired by Refs. [36–38], we removed the daily jumps of the stock index due to the difference between the closing and opening index. In KOSPI, there was no trading after closing and before opening the market until 1995. So, the changes of the index associated with the daily jumps are small during those years. Since 1995, after-market trading and before-market trading have been allowed for one hour each, leading to substantial overnight