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Diffusion entropy analysis on the scaling behavior of financial markets

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Abstract

In this paper the diffusion entropy technique is applied to investigate the scaling behavior of financial markets. The scaling behaviors of four representative stock markets, Dow Jones Industrial Average, Standard&Poor 500, Heng Seng Index, and Shang Hai Stock Synthetic Index, are almost the same; with the scale-invariance exponents all in the interval $[0.92, 0.95]$. We also estimate the local scaling exponents which indicate the financial time series is homogenous perfectly. In addition, a parsimonious percolation model for stock markets is proposed, of which the scaling behavior agrees with the real-life markets well.

Keywords: Diffusion entropy analysis; Financial market; Artificial stock market

1. Introduction

Analysis of financial time series attracts special attentions from diverse research fields for several decades. It cannot only reveal the intrinsic dynamical properties of the corresponding financial markets but also provide us a clear scenario to construct dynamical models. Traditional theories are constructed based upon some basic hypothesis, to cite examples, the stochastic processes in the markets and the homogenous property of the markets. The unexpected so-called rare events are explained simply as the results due to accidents or external triggers. The advancements in nonlinear theory lead a complete revolutionary in our ideas about financial markets [1]. Instead of the deduced Gaussian distribution, empirical investigations in recent years indicate that the price return distribution of the financial time series generally obeys the centered Lévy distribution and displays fat-tail property, and the financial time series exhibits the scale-invariance behavior [2–7]. The nonlinear theory based analysis and dynamical models for the financial markets are the essential problems at present time.

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One of the important features of the financial time series is the scale-invariance property, which can highlight the dynamical mechanics for the corresponding markets. Consider a complex system containing a large amount of particles. The scale-invariance property in the diffusion process of this system can be described mathematically with the probability distribution function as

$$P(x,t) = \frac{1}{t^{\delta}} F\left(\frac{x}{t^{\delta}}\right), \tag{1}$$

where x is the displacements of the particles in the complex system and δ the scale-invariance exponent. The theoretical foundation of this property is the central limit theorem and the generalized central limit theorem [8,9]. For the standard diffusion process, the parameter is $\delta = 0.5$ and $F(y)$ is the Gaussian function. And $\delta \neq 0.5$ exhibits the deviation of the dynamical process from the normal stochastic one. For a financial time series, the delay−register vectors, denoted with $\{y_k, y_{k+1}, \ldots, y_{k+m-1} | k = 1, 2, 3, \ldots, N-m+1\}$, can be regarded as the trajectories of $N-m+1$ particles during the period of 0 to m. By this way we can map a time series to a diffusion process, called overlapping diffusion process in this paper. An alternative solution is to separate the considered time series into many non-overlapping segments and regard these segments as the trajectories.

In literature, several variance-based methods are proposed to detect the scale-invariance properties, such as the probability moment method [10], the fluctuation approach, the de-trended fluctuation approach [11], the factorial moments [12–14] and the de-trended moving average [15]. However, these variance-based methods have two basic shortcomings. One is the scale-invariance property can be detected, while the value of the exponent may not be obtained correctly. The other is for some processes, like the Lévy flight, the variance tends to infinite and these methods are unavailable at all. Although the infinite cannot be reached due to the finite records of empirical data, clearly we cannot obtain correct information about the dynamics under these conditions. Hence, a new method called diffusion entropy analysis (DEA) is proposed to overcome these problems [16]. In this paper the DEA is used to detect the scaling behavior of financial markets.

2. Diffusion entropy technique and data analysis

As mentioned in Section 1, to overcome the above shortcomings in the variance-based methods, the authors in Ref. [16] proposed the DEA. To keep our description as self-contained as possible, we review the DEA method briefly.

Filter out the trends in the original time series. Adopting the traditional assumption generally used in the research filed of engineering, that a discrete time series variable consists of a slowly varying part and a fluctuation part [17,18], the index of a stock market reads,

$$\xi_j = S_j + \zeta_j, \quad j = 1, 2, \ldots, N, \tag{2}$$

where ζ_j is the fluctuation with zero mean and fixed variance.

In the signal processing, the slow and regular variation S_j is usually called signal, which contains the useful information. And the rapid erratic fluctuation ζ_j is called the noise, which is regarded as perturbations containing only trivial information. In the DEA method, however, the scale-invariance will be detected from this fluctuation part. The "step smoothing" [17] procedure is employed to estimate the S_j part in our calculations, that is, regard the average of the segments as the trends, respectively. The final time series is regarded as a steady series, whose overlapping diffusion process reads,

$$x_k(t) = \sum_{j=k}^{k+t} \zeta_j, \quad k = 1, 2, \ldots, N-t+1, \tag{3}$$

herein, $x_k(t)$ is defined as kth trajectories by the superposition of these fluctuations ζ_j, and these trajectories generate a diffusion-like process. Consequently, the Shannon entropy can be defined as,

$$S(t) = -\int_{-\infty}^{+\infty} P(x,t) \log_{10}[P(x,t)]\, \mathrm{d}x. \tag{4}$$