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Many-to-one matching markets with externalities among firms[☆]

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ABSTRACT

We study a labor market described by a many-to-one matching market with externalities among firms in which each firm's preferences depend not only on workers whom it hires, but also on workers whom its rival firms hire. We define a new stability concept called weak stability and investigate its existence problem. We show that when the preferences of firms satisfy an extension of substitutability and two new conditions called increasing choice and no external effect by an unchosen worker, then a weakly stable matching exists. We also show that a weakly stable matching may fail to exist without these restrictions.

1. Introduction

Since the seminal paper of Gale and Shapley (1962), matching markets have been extensively analyzed by many researchers. We refer to Roth and Sotomayor (1990) for a detailed study of the literature. A typical application of matching markets is a labor market. Such a market is often formulated by a many-to-one matching market in which each firm can hire multiple workers, and each worker can work for at the most one firm (cf. Kelso and Crawford, 1982, Echenique and Oviedo, 2004 and Hatfield and Milgrom, 2005). Their studies focus on stable matchings in which firm and workers cannot deviate profitably. In standard models, it is assumed that each firm's preferences depend only on workers it hires. In a labor market, however, it is natural to consider instances in which each firm's preferences depend on workers whom its rival firms hire, since they compete among themselves in a market. This paper takes the externalities among firms into account.[1] We define a new stability concept called weak stability, and provide restrictions on the preferences of firms which guarantee the existence of a weakly stable matching.

There are several studies on one-to-one matching markets with externalities. Sasaki and Toda (1986) first introduced one-to-one matching markets with externalities. When externalities are present, the stability of a matching depends on how deviating agents predict the reaction of the other agents. They considered three stability concepts, and investigated their existence in general one-to-one matching markets.[2] Sasaki and Toda (1996) generalized the results of Sasaki and Toda (1986) by using what they define as an estimation function, which describes the set of possible matchings that the deviating pair predicts will result from their deviation. Given such estimation functions, a pair of agents deviates from a matching, if they are made better off under all matchings predicted by their estimation functions. They showed that a stable matching exists in all markets if and only if each agent has a universal estimation function which considers all matchings possible. This result is due to the assumption that estimation functions are given exogenously. Hafalir (2008) introduced an endogenous estimation function which depends on preferences of the other agents. He provided a sufficient condition for estimation functions to be compatible with the existence of a stable matching, and showed that a particular notion of endogenous beliefs called sophisticated expectations guarantees the existence of a stable matching. Mumcu and Saglam (2010), on the other hand, considered P-stability of Sasaki and Toda (1986),

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1 For workers' externalities, Dutta and Masso (1997), Ma (2001), Echenique and Yenmez (2007) and Pycia (forthcoming) analyzed a many-to-one matching market with peer effects.

2 They defined "stability", "P-stability and "O-stability". A stable matching is defined on the assumption that a pair of agents deviates from a matching, anticipating the worst matching for them. In P-stability, a pair of agents deviates, keeping the other agents' matchings fixed. In O-stability, a pair of agents deviates, anticipating the most desirable matching for them. They showed that a stable matching always exists, while the other solutions may be empty for some matching market.

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which is defined on the assumption that a pair of agents deviates from a matching, keeping the other agents' matchings fixed. They provided some restrictions on preferences for the existence of a P-stable matching.

Our study builds on Mumcu and Saglam (2010) by considering a many-to-one market. We first analyze a small market by using a strong stability which is the many-to-one analogue of the solution concept used in Mumcu and Saglam (2010). A strongly stable matching may not always exist even in a small market due to incredible deviations. A deviation can be regarded as incredible when the deviation of a firm and workers yields a further deviation within the deviating members and workers who are hired by the deviating firm before the deviation. We introduce the notion of "strongly blocking" to rule out incredible deviations, and define a weakly stable matching that cannot be strongly blocked by any pair.[3]

The set of weakly stable matchings is nonempty under a certain condition on the external effect. We show that a weakly stable matching exists if the preferences of firms satisfy an extension of substitutability and two new conditions called increasing choice and no external effect by an unchosen worker. Substitutability is originally introduced by Kelso and Crawford (1982) for the matching model without externalities, and is a sufficient condition for the existence of a stable matching. An extension of substitutability for our model, however, does not guarantee the existence of a weakly stable matching in a market with externalities.

Increasing choice requires that (i) the choice set of a firm depends only on the set of workers hired by its rival firms, and (ii) the choice set of a firm expands when the set of workers hired by the rival firms expands. No external effect by an unchosen worker means that if firm f does not choose worker w from a subset of workers, then firm f's choice from another subset of workers in which worker w is excluded is not affected by a rival firm additionally hiring worker w. In other words, an external effect in firm f's choice is caused only by an important worker for firm f. We show, by example, that a weakly stable matching may not exist without increasing choice or no external effect by an unchosen worker.

By imposing an additional assumption, we also give a sufficient condition for the existence of a strongly stable matching. The sufficient condition is substantially different from that of Mumcu and Saglam (2010), because applying their condition to our model restricts not only on the preferences of firms, but also those of workers.

The rest of this paper is organized as follows. Section 2 introduces the model of a many-to-one matching market with externalities among firms and defines two stability concepts: strong stability and weak stability. In this section, restrictions on preferences for firms are also discussed. In Section 3, a relationship between the strong and weak stability is discussed and the existence of a weakly stable matching is proved. Section 4 concludes.

2. Model

2.1. Many-to-one matching market with externalities among firms

Let $F = \{f_1, \ldots, f_m\}$ be the set of m firms and $W = \{w_1, \ldots, w_n\}$ be the set of n workers. F and W are disjoint sets. We assume that each firm can hire multiple workers, but each worker is allowed to work for at the most one firm. A matching μ is a function from $F \cup W$ into $2^{F\cup W}$ such that for all $w \in W$ and all $f \in F$, (i) $\mu(w) \in 2^F$ and $|\mu(w)| \leq 1$, (ii) $\mu(f) \in 2^W$ and (iii) $\mu(w) = \{f\}$ if and only if $w \in \mu(f)$. Let $M(F, W)$ be the set of matchings. We often regard a matching μ as a vector $(\mu(f_1), \ldots, \mu(f_m))$.

Each worker w has a strict, transitive and complete preference relation $\succeq_w$ over $F \cup \{\emptyset\}$. Each firm f has a strict, transitive and complete preference relation $\succeq_f$ over $M(F, W)$, because externalities exist among firms.

Let $\mu \in M(F, W), f \in F$ and $C \subseteq W$. Define $\mu^{f,C}$ as a matching such that

if $w \in C$, then $\mu^{f,C}(w) = \{f\}$,
if $w \in \mu(f) \setminus C$, then $\mu^{f,C}(w) = \emptyset$,
if $w \notin C \cup \mu(f)$, then $\mu^{f,C}(w) = \mu(w)$.

In other words, $\mu^{f,C}$ is a matching in which firm f hires C and fires $\mu(f) \setminus C$, keeping the other agents' matchings fixed.

Let $\mu \in M(F, W)$ and $C \subseteq W$. Define μ^C as a matching such that

if $w \in C$, then $\mu^C(w) = \emptyset$,
if $w \notin C$, then $\mu^C(w) = \mu(w)$.

That is, μ^C is a matching in which all workers of C quit a job at μ, keeping the other agents' matchings fixed.

We define the choice set of a firm below. Let $f \in F$ and $C \subseteq W$. Define

$R(f, C) = \{\mu \in M(F, W) | \mu(f) = \emptyset \text{ and } \mu(w) = \emptyset \text{ for all } w \in C\}$,

which is the set of matchings where all workers in C are unemployed and firm f does not hire any workers. For each $\mu \in R(f, C)$, the choice set $Ch_f(C|\mu)$ is a firm f's most preferred subset of C given μ; that is, $Ch_f(C|\mu)$ is the set such that (i) $Ch_f(C|\mu) \subseteq C$ and (ii) $\mu^{f,Ch_f(C|\mu)} \succeq_f \mu^{f,C'}$ for all $C' \subseteq C$. When externalities are present, the choice set of a firm may depend on how the other firms are matched.

2.2. Stability concepts

In this subsection, we introduce two stability concepts; strong stability and weak stability. These are defined on the assumption that a firm and workers deviate from a matching, keeping the other agents' matchings fixed.

A matching μ is individually rational for workers if $\mu(w) \succeq_w \emptyset$ for all $w \in W$, and for firms if $Ch_f(\mu(f)|\mu^{\mu(f)}) = \mu(f)$ for all $f \in F$. We say that a matching μ is individually rational if it is individually rational for workers and firms. When a matching is individually rational, each worker has no incentive to quit a job, and each firm has no incentive to fire a subset of its workers.

A matching μ is weakly blocked by $(f, C) \in F \times 2^W$ if (i) $C \setminus \mu(f) \neq \emptyset$, (ii) $\mu^{f,C} \succ_f \mu$ and (iii) $f \succeq_w \mu(w)$ for all $w \in C$. Condition (i) requires that C contains at least one worker who is not hired by firm f at μ. Condition (ii) and (iii) mean that given μ, firm f is strictly better off by hiring workers C, and all workers in C are better off by working at f. A matching μ is strongly stable if it is individually rational and is not weakly blocked by any pair. Let SS denote the set of strongly stable matchings.

The strong stability is an extension of the pairwise stability by Mumcu and Saglam (2010).[4] They showed that if there are at least three agents, we can construct preferences of agents in which a strongly stable matching does not exist. A reason for the nonexistence is that a strong stability admits incredible deviations. A deviation by a firm and a set of workers is incredible if that deviation then set off a further deviation within the deviating

[3] Note that our stability concept is different from the weak stability introduced by Klijn and Masso (2003) in one-to-one matching markets without externalities.

[4] They also consider the core stability which does not admit any coalitional deviations.