Available online at www.sciencedirect.com
SCIENCE DIRECT®
Games and Economic Behavior 48 (2004) 1–17
GAMES and Economic Behavior
www.elsevier.com/locate/geb

Restabilizing matching markets at senior level

David Cantala

Escuela de Economía, Universidad de Guanajuato, Calzada de Guadalupe s/n CP 36 000, Guanajato Gto, Mexico

Received 6 February 2001

Available online 6 November 2003

Abstract

We study, in many-to-one matching markets, the restabilization process of a group-stable matching disrupted by a change in the population. If firms are not allowed to fire workers, the market always reaches stability again only if the disruption is due to the opening of positions by firms, or the retirement of workers. This is shown by designing an algorithm which always leads to a group-stable matching. The algorithm mimics markets where firms make offers and workers accept the offer of their favorite firm. If firms are allowed to fire workers, we construct another algorithm which produces a group-stable matching when the disruption is due to the entrance of workers or the closure of positions. In this algorithm, unemployed workers make offers to firms. In both cases of disruption, we require firms to have *q-substitutable* preferences.

JEL classification: C78; J63

Keywords: Stable matchings; Senior level markets; Deferred acceptance algorithm

1. Introduction and preliminaries

In markets at senior level, firms and workers are matched to one another and stable matchings are disrupted by changes in the population of workers, or in the set of positions provided by firms. Such disruptions trigger dynamics of offers and acceptances different from the ones observed on junior level markets. In particular, vacancies are filled sequentially, not in cohort. We consider that firms may hire many workers, have *q-substitutable* preferences (see below) and study the restabilization process in senior markets, initially considering the case in which workers may have a tenure. We design an

E-mail address: dcantala@quijote.ugto.mx.

0899-8256/$ – see front matter
doi:10.1016/j.geb.2003.07.005

algorithm, where unmatched firms make offers, which always leads to a stable matching whenever the disruption is due to the opening of positions or the retirement of workers. Then, when firms are allowed to fire workers, we show that an algorithm where offers are processed by unemployed workers, restabilizes any matching arising from the disruption of a stable matching due to the entrance of workers or the closure of positions.

Formally, consider two disjoint and finite sets of agents: $\mathcal{F} = \{f_1, \ldots, f_m\}$, the set of firms, and $\mathcal{W} = \{w_1, \ldots, w_n\}$, the set of workers. The problem, introduced by Gale and Shapley (1962), consists in assigning workers to firms. We denote by $q = (q_f)_{f\in\mathcal{F}}$ the vector of quotas associated with each firm; more specifically q_f is the maximum number of workers that can be assigned to firm f. A matching μ is a mapping from the set $\mathcal{F} \cup \mathcal{W}$ into the set of all subsets of $\mathcal{F} \cup \mathcal{W}$, such that for all $f \in \mathcal{F}$ and $w \in \mathcal{W}$:

(1) $\mu(f) \in 2^{\mathcal{W}}$ and $|\mu(f)| \leqslant q_f$, i.e., a firm will be either matched to a subset of workers whose cardinality is at most equal to its quota or it will remain single;
(2) either $|\mu(w)| = 1$ and $\mu(w) \in \mathcal{F}$, or $\mu(w) = \emptyset$, i.e., a worker is either matched to a firm or is single; and
(3) $\mu(w) = f$ if and only if $w \in \mu(f)$, namely the relationship is reciprocal.

Each firm f has a strict, transitive and complete preference relation $\succ_f$ over the family of subsets of workers $2^{\mathcal{W}}$. We interpret the empty set as firm f not being assigned to any worker. When a firm ranks the empty set better than a subset, it means that it prefers remaining single than being assigned to this subset. Given a set $S \subseteq \mathcal{W}$, let the *choice* of firm f, denoted $Ch(S, q_f, \succ_f)$, be f 's most preferred subset of S with cardinality at most q_f according to its preference ordering $\succ_f$. Each worker w has a strict, transitive and complete preference relation $\succ_w$ over the set $\mathcal{F} \cup \emptyset$. We interpret the empty set as w being unemployed. Preference profiles are $(m+n)$-tuples of preference relations and they are represented by $\succ = (\succ_{f_1}, \ldots, \succ_{f_m}, \succ_{w_1}, \ldots, \succ_{w_n})$. A matching market is a quadruple $(\mathcal{F}, \mathcal{W}, q, \succ)$.

The relevant criteria in such markets is the stability of matchings. In order to specify the stability concepts required in our analysis, the following definitions are introduced. Let μ be a matching, then we denote $\mathcal{W}_{f,\mu}$ the set of workers who prefer firm f to their match under μ; formally, $\mathcal{W}_{f,\mu} = \{w \in \mathcal{W} \mid f \succ_w \mu(w)\}$. We say that a matching μ is *blocked by* a worker w if $\emptyset \succ_w \mu(w)$; i.e., if she prefers remaining alone to being matched to $\mu(w)$. Similarly a matching μ is *blocked by* a firm f if $\mu(f) \neq Ch(\mu(f), q_f, \succ_f)$; i.e., if the firm wishes to fire some or all its matches at μ. A matching μ is *blocked by* a worker-firm pair (w, f) if $f \succ_w \mu(w)$ and $Ch(\mu(f) \cup \{w\}, q_f, \succ_f) \succ_f \mu(f)$; i.e., worker w prefers f to her match and f wishes to hire w. Finally a matching μ is *blocked by a pair (S, f)* with $S \subseteq \mathcal{W}$ and $f \in \mathcal{F}$, if $S \neq \emptyset$, $S \subseteq \mathcal{W}_{f,\mu}$ and $S \subseteq Ch(\mu(f) \cup S, q_f, \succ_f)$; i.e., if all workers w in S prefer being matched to firm f than to $\mu(w)$ and f would like to hire S. We say that (S, f) forms a blocking coalition of μ.

A matching μ is *individually worker rational* (IWR) if it is not blocked by any worker and *individually firm rational* (IFR) if it is not blocked by any firm. Finally a matching is *individually rational* (IR) if it is individually worker rational and individually firm rational. We denote by $A(\mathcal{F}, \mathcal{W}, q, \succ)$ the set of worker-firm coalitions individually rational, or acceptable to each member of the respective coalitions of market $(\mathcal{F}, \mathcal{W}, q, \succ)$.