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A note on cointegration of international stock market indices

Thomas Dimpfl *

University of Tübingen, Faculty of Economics and Social Sciences, Mohlstr. 36, 72074 Tübingen, Germany

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ABSTRACT

Cointegration is frequently used to assess the degree of interdependence of financial markets. We show that if a stock's price follows a stock specific random walk, market indices cannot be cointegrated. Indices are a mere combination of *n* different random walks which itself is non-stationary by construction. We substantiate the theoretical propositions using a sample of 28 stock indices as well as a simulation study. In the latter we simulate stock prices, construct indices and test whether these indices are cointegrated. We show that while heteroscedasticity misleads cointegration tests, it is not sufficient to explain the high correlation between stock market index returns. A common random walk component and correlated price innovations are necessary to reproduce this feature.

1. Introduction

Building on the time series properties of stock market index data, a vast literature has emerged which studies the dependence and the degree of integration of international financial markets by means of cointegration analysis. Due to the relatively strong comovement of financial markets, the assumption of a shared common trend seems plausible at first sight. This is the reason why cointegration analysis has been a major tool in the study of interrelations between financial markets. However, there are two major issues that have to be taken into account.

First, the cointegration relationship seems to be a very fragile one. Different studies using the same indices do not necessarily find an identical number of cointegrating vectors. Most of the studies are conducted in the spirit of Kasa (1992) who can identify one common stochastic trend for the stock markets of the USA, Japan, United Kingdom, Germany, and Canada. He uses monthly and quarterly data over a period of almost 16 years which suits the notion that cointegration is a long term concept while short run deviations from the common trend are possible. As opposed to these findings, Pascual (2003) finds no cointegration relationship between the French, German, and UK stock market using quarterly data for an even longer sample from 1960 to 1999. Statistically, if the time series are not found to be cointegrated in the larger sample, they should not be found to be cointegrated on any subsample like the one employed by Kasa (1992). There are numerous further examples in the literature where a slight alteration of the approach leads to different results. For example, Aggarwal and Kyaw (2005) and Phengpis and Swanson (2006) both investigate the NAFTA countries. While Aggarwal and Kyaw (2005) find evidence for cointegration in the post-NAFTA era, Phengpis and Swanson (2006) do not. Detecting cointegration, thus, seems to critically depend upon the time span under consideration and the precise specification of the statistical model.

The second issue is that Johansen's (1988) test for cointegration—the major tool in empirical work—is prone to misjudgement. Financial data are marked by heteroscedasticity which is known to bias the test (cp. Lee & Tse, 1996). Also, in particular in early studies like Kasa (1992), a small sample size has been a major issue. Even though accounting for heteroscedasticity (e.g. Cavaliere, Rahbek, & Taylor, 2010) and small sample size is possible (cp. Barkoulas & Baum, 1997; Johansen, 2002), it is hardly ever done. Recent studies, however, use in general daily data so that at least the latter issue can be regarded as overcome.

The fact that cointegration among stock market indices is a delicate issue has first been addressed by Richards (1995). He relies on the Capital Asset Pricing Model (CAPM) for stock prices and shows that indices, constructed as weighted averages of stock prices in a country, cannot be cointegrated. We contribute to the literature by translating his argument to international stock markets. In addition, we abandon the CAPM assumption in favour of the more flexible random walk model for stock prices, pursuing a twofold aim: Building on theoretical and statistical arguments, we demonstrate that stock market indices cannot be cointegrated. Furthermore, we foster an intuition for the heterogeneous results found in the literature.

The argumentation is based on three pillars. First, we use the random walk model for stock prices to derive that statistically cointegration

* Tel.: +49 70 71 29 76 417; fax: +49 70 71 29 55 46.
E-mail address: thomas.dimpfl@uni-tuebingen.de.

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between two markets is impossible. In our setting, stock market indices are a weighted average of random walk asset prices. These individual stochastic trends never cancel out in a cointegration regression and consequently prevent the indices from being cointegrated. Second, we perform an empirical exercise to show that using standard methodology cointegration is very unstable and that the results are at odds with the notion of long term comovement. In particular, we employ subsamples and show that detection of cointegration among pairs of stock market indices is basically random. And third, we simulate the theoretical model to gain an idea of the components of stock market indices. The aim of the simulation study is to reproduce data series which exhibit similar properties as the observed stock market indices in a cointegration framework.

We document that in a cointegration analysis of international stock market indices, every desired outcome can be produced by suitably restricting the sample period and adjusting the model. Standard cointegration tests will reject the null hypothesis of no cointegration far too often if the properties of the data (in particular heteroscedasticity) are disrespected. Our simulation study suggests that most likely stock market indices share an additional common stochastic trend and that returns are correlated. However, cointegration analysis is not a suitable methodology to investigate the comovement of stock markets if individual stocks are random walks themselves.

The paper proceeds as follows. Section 2 reviews the related literature and points out the critical issues raised in the Introduction. Section 3 outlines the random walk model of stock prices and highlights the implications for stock indices and cointegration. Section 4 presents the results of a cointegration analysis of 28 stock market indices in order to highlight inconsistencies when applying the cointegration methodology to empirical data. Section 5 holds a simulation study of the theoretical model and Section 6 concludes.

2. Related literature

Based on the assumption that stock markets in different countries share common stochastic trends, numerous studies have tried to detect those. One of the first was Kasa (1992) who can identify one common stochastic trend for the stock markets of the U.S., Japan, England, Germany, and Canada. He used monthly and quarterly data over a period of almost 16 years. More recent contributions include Choudhry, Lu, and Peng (2007), Lagoarde-Segot and Lucey (2007) and Valadkhani and Chancharat (2008). These studies have in common that they all identify exactly one common stochastic trend. However, there is no economic or financial theory predicting the number of common stochastic trends. Empirically, Click and Plummer (2005), for example, investigate the relationship between five ASEAN stock markets on a daily basis for four years and find that these markets are cointegrated. However, the authors can identify only one cointegrating vector. This implies four stochastic trends which influence the cointegration relationship. The authors conclude that in this case the integration of these financial markets is far from being perfect. Empirical work, thus, cannot unambiguously deduce the number of stochastic trends shared by financial markets. The number of detected cointegrating vectors critically depends on the number of markets analysed, the sample time span, data frequency and the properties of financial data like fat tails or heteroscedasticity.

Empirical evidence is not only mixed with respect to the number of common trends. The question whether financial markets share a common stochastic trend at all is also not answered unambiguously. The studies cited above find evidence for the existence of a cointegration relationship. In contrast, Chan, Gup, and Pan (1997) who analyse 18 stock market indices, find that these markets are not cointegrated. The analysis is conducted using monthly data from 1961 to 1992. Pascual (2003) studies whether the degree of integration between the French, German, and UK stock markets increases. He does not find a cointegration relationship using quarterly observations from 1960 to 1999 either. The results of Narayan and Smyth (2005) who investigate the relationships between the stock markets of New Zealand, Australia and the G7 countries, are mixed, depending on which test they use to detect cointegration. Their analysis is based on real monthly observations from 1967 to 2003.

With respect to financial theory, the existence of cointegration relationships would contradict the Efficient Market Hypothesis (EMH) which requires that returns—and with them future prices—are not predictable in the long run.[1] A common model, frequently used in the literature, which captures this behaviour of stock or index returns at high frequencies, is the random walk model for stock prices. It dates back to work by Fama (1965) and Malkiel (1973) and has ever since frequently been applied (see, inter alia, Black, 1986; Godfrey, Granger, & Morgenstern, 2007) and tested, albeit with mixed results (see, inter alia, De Bondt & Thaler, 1985; Fama, 1995; Worthington & Higgs, 2009). Cointegration, by contrast, would allow for some kind of predictability in the long run, even though short run predictions are not possible. This argument is not limited to stock markets. Granger (1986) shows that gold and silver prices are not cointegrated once these prices are generated on an efficient market. The very same is true for stock prices. That cointegration based analysis of market efficiency is unreliable has then been pointed out by Barkoulas and Baum (1997) in the context of foreign exchange markets.

This paper suggests that under the assumption that stock prices are generated according to the random walk model, international financial markets are not cointegrated in the Engle and Granger (1987) sense. For the most part, we follow arguments that have been put forward by Richards (1995) who claims that stock return indices in one stock market cannot be cointegrated if one assumes that excess returns are generated according to the CAPM. He argues that in order to be cointegrated, the company specific shocks of one company need to offset the shocks of the other company. However, both of these shocks would have to be completely unexpected, but identical in size and direction. He states that this would rule out the possibility that any management decision permanently affected a company's stock price. He summarises that these company specific shocks "will not translate into a cointegrating relationship between the actual return indices for the two (or more) assets." It seems that this result has been neglected in some of the literature on cointegration of financial markets since then. This paper will therefore reinforce the argumentation that company specific shocks eventually inhibit the existence of cointegration relations (as defined by Engle & Granger, 1987) between international stock market indices. In contrast to Richards (1995) who seeks to explain the results of Kasa (1992) obtained on low frequencies, our line of argumentation will keep features of high frequency data in mind. Our model will therefore be different from Richards (1995) in that we will not rely on the CAPM, but the more general random walk model for stock prices. It is widely accepted that on high frequencies stock prices are modelled best by a random walk. Further, Richards (1995) attributed some of the results in the literature specifically to a small sample bias in the Johansen (1988) cointegration testing framework. This issue can be regarded as overcome since high frequency data (in particular daily data) are nowadays widely (and even freely) available. However, daily data are marked by other features (e.g. heteroscedasticity) which have to be taken into account when testing international financial markets for cointegration.

3. Stock prices, indices, and cointegration

The basic model for stock prices which is widely used in the literature, assumes that log-prices *individually* follow a random walk. The model can be written as

$$p_{i,t} = p_{i,t-1} + e_{i,t}, \tag{1}$$

[1] Short-run predictions may be possible due to market frictions and investor behaviour.

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