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On market-inspired approaches to propositional satisfiability ☆

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Abstract

We describe three market-inspired approaches to propositional satisfiability. The first is based on a formulation of satisfiability as production on a supply chain, where producers of particular variable assignments must acquire licenses to fail to satisfy particular clauses. Experiments show that although this general supply-chain protocol can converge to market allocations corresponding to satisfiable truth assignments, it is impractically slow. We find that a simplified market structure and a variation on the pricing method can improve performance significantly. We compare the performance of the three market-based protocols with distributed breakout algorithm and GSAT on benchmark 3-SAT problems. We identify a tradeoff between performance and economic realism in the market protocols, and a tradeoff between performance and the degree of decentralization between the market protocols and distributed breakout. We also conduct informal and experimental analyses to gain insight into the operation of price-guided search.

Keywords: Satisfiability; Market-oriented programming; Distributed constraint satisfaction; Auctions

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1. Introduction

Multiple agents must often engage in activities with complex, interrelated dependencies. These dependencies may arise from contention for limited resources, scheduling constraints, or direct dependencies of activity on the results of other agents' activities. Often, such problems of choosing activities, allocating resources, determining schedules, and composing results from individual agents' actions can be modeled as constraint satisfaction problems (CSPs).

As members of the class of NP-complete problems, CSPs are widely considered to be intractable. Even the best algorithms for NP-complete problems generally require exponential time in the worst case. The problem of solving CSPs in the multiagent context is further complicated by their decentralized nature, which imposes constraints on the distribution of information and authority among participants. In a ***decentralized system***, the information state of an individual is considered private, and is disseminated only by voluntary communication acts. This contrasts with centralized systems, in which it is generally assumed that a single entity (the "center") can obtain knowledge of the entire information state, for example by compelling communication. Decentralization constraints clearly restrict the computations performed by individual participants.

Yokoo and Hirayama [34,36] formalize CSPs with decentralization constraints as distributed constraint satisfaction problems (DisCSPs) and, with others, have designed a variety of effective algorithms, such as the distributed breakout (DB) algorithm [35]. These approaches are generally distributed adaptations of centralized algorithms, and can perform very effectively.

Markets provide another model of decentralized systems with clearly delineated boundaries of knowledge and lines of communication. Typically, participants (agents) maintain knowledge of only resources of direct interest, and interact with other agents only indirectly through market institutions, such as auctions. The market-based approach has become increasingly popular in recent years, as evidenced by the growing prevalence in the AI literature of research in the design and analysis of computational market systems and their underlying mechanisms. Markets have been proposed to solve a diversity of problems, with climate control [33], power load management [32], allocating computational servers [22], multicommodity flow [29], and belief aggregation [12] being just a sample.

Shoham and Tennenholtz [19] directly pose the question "What can a market compute, and at what expense?" They provide answers for some interesting cases, applying concepts from economic mechanism design and communication complexity. Different behavioral assumptions can support conclusions about additional cases. For instance, over fifty years ago, Samuelson [13] considered how markets could decentralize the solution of linear programming problems. More generally, adopting market protocols in the framework of general equilibrium theory can be seen to yield a computational model capable of solving convex programming problems [3]. However, none of these lines of analysis provide answers with respect to the sort of combinatorial optimization problems of most interest in AI and multiagent systems research.

To address this gap, we examine here the possibility of using markets to solve propositional satisfiability (SAT) problems in a decentralized manner. We refer to a market