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Non-extensive behavior of a stock market index at microscopic time scales

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Abstract

This paper presents an empirical investigation of the intraday Brazilian stock market price fluctuations, considering q-Gaussian distributions that emerge from a non-extensive statistical mechanics. Our results show that, when price returns are measured over intervals less than one hour, the empirical distributions are well fitted by q-Gaussians with exponential damped tails. Scaling behavior is also observed for these microscopic time intervals. We find that the time evolution of the return distributions is according to a super-diffusive q-Gaussian stationary process within a nonlinear Fokker–Planck equation. This regime breaks down due to the exponential fall-off of the tails, which in turn, governs the transient dynamics to the long-term macroscopic Gaussian regime. This exponentially damped, non-extensive modeling provides a new framework to investigate the dynamics of other stock markets intraday price fluctuations.

Keywords: Non-extensive statistical mechanics; Stochastic processes; Econophysics; Non-linear dynamics; High-frequency returns

1. Introduction

The empirical probability distribution functions (PDFs) of price fluctuations of financial indices for different markets have been reported in the econophysics literature in recent years [1,2]. Although there has been a huge progress in the statistical description of these fluctuations, a complete and consistent description of the distributions and its dynamics in the high-frequency regime is still lacking.

It is well known that high-frequency financial time series such as foreign exchange rates or stock market returns are long-memory non-Gaussian processes. Such complex behavior calls for advanced theories in the field of statistical physics. Whence, we consider the recently proposed non-extensive statistical physics [3], a theory that generalizes the extensive classical statistical theory for strong dependent variables. A large number of studies [4–7] of financial markets have already employed the non-extensive statistics in their analysis.

In this paper, we investigate the intraday dynamics of the BOVESPA index, the financial index of the Brazilian stock market, one of the largest emerging markets in the world. We model the distributions of the index price fluctuations by q-Gaussians, which are a class of stable distributions that emerge from the

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non-extensive approach, where the parameter q measures the degree of the non-extensivity of the stochastic process.

We find that for time horizons less than one hour, the probability distributions of price fluctuations are well fitted by q-Gaussians with damped exponential tails. A q-Gaussian scaling at the center of the distributions with $q^* = 1.75$ is observed, due to the presence of strong correlations between price fluctuations. The quasi-stationary q^*-Gaussian regime breaks down due to the exponentially truncated tails and a new intraday correlated regime emerges at larger time scales, in which the q^*-Gaussian central part of the distributions is consumed by the exponential tails.

The dynamics in the short-time regime is also according to a super-diffusive q-Gaussian stationary process governed by a non-linear Fokker–Planck equation (FPE). Therefore, a coherent description encompassing the non-extensive statistics is addressed, in which the high-frequency price fluctuations are modeled by a correlated anomalous diffusion process.

This paper is organized as follows. In Section 2, we describe the empirical observations for the high-frequency BOVESPA index. In Section 3, we present the non-extensive statistical theory. In Sections 4 and 5, these observations are analyzed according to the non-extensive approach. In Section 6, we present some concluding remarks.

2. Empirical results

Our results are based on the analysis of BOVESPA index high-frequency 30 s interval data collected from November 2002 to June 2004, a period without major local market disturbances. These data consist of a set of 352 500 prices (index values) $p(t)$ which allow a fairly statistical analysis.

From this database, we select a complete set of non-overlapping price fluctuations (returns):

$$x_\tau(t) = \ln p(t+\tau) - \ln p(t) \tag{1}$$

occurring in τ-intraday interval. We define the normalized returns as

$$x_\tau^N(t) = \frac{(x_\tau(t) - \mu_\tau)}{\sigma_\tau}, \tag{2}$$

where μ_τ and σ_τ are, respectively, the mean and the standard deviation of $x_\tau(t)$.

To characterize quantitatively the observed stochastic process, we measure the probability distribution $P(x_\tau^N)$ of index price fluctuations for τ ranging from 1 to 60 min. The number of data in each set decreases from the maximum value of 176 250 ($\tau = 1$ min) to the minimum value of 29 375 ($\tau = 60$ min).

In Fig. 1 we show the measured (non-normalized) probability distribution for $\tau = 1$ min normalized returns and compare with the expected distribution for the Gaussian process. The empirical distribution is clearly fat tailed, indicating that non-Gaussian time series analysis is required.

The scaling behavior of the distributions at coarser time scales has different regimes. At microscopic scales (typically shorter than one hour), correlations between successive price changes are dominant due to informational flow among the financial traders. On the other hand, at macroscopic scales (typically larger than one month) macroeconomic rules govern the market drift and correspond to a Gaussian regime. Mesoscopic time scales can be associated with a transient regime between the micro and macro regimes.

The evolution of the PDFs at microscopic and mesoscopic time scales is still an open question. In this paper, we investigate the scaling behavior of the return distributions at microscopic time scales. As shown in Fig. 2, the data collapse into the $\tau = 1$ min distribution, but this stationarity breaks down at $\tau = 1$ h. This means that the $\tau = 1$ min return distribution functional form is preserved across the microscopic scales, therefore exhibiting a short-time non-Gaussian intraday quasi-stationary regime.

We also investigate the presence of linear and non-linear dependence between successive $\tau = 1$ min normalized returns. In Fig. 3(a) we show the linear autocorrelation $R(\Delta t) = \langle x_\tau^N(t) x_\tau^N(t+\Delta t)\rangle$ according to the time lag Δt of the measured returns. Employing the standard least-squares fit for time intervals $\tau = 2$–15 min, the semi-log plot indicates a short-range exponential behavior $R(\Delta t) \sim \exp(-\Delta t/\Gamma)$ with characteristic time scale $\Gamma \cong 4$ min. After $\Delta t \cong 20$ min the correlation is at the noise level.