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Fractional integration in daily stock market indexes

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Abstract

I use parametric and semiparametric methods to test for the order of integration in stock market indexes. The results, which are based on the EOE (Amsterdam), DAX (Frankfurt), Hang Seng (Hong Kong), FTSE100 (London), S&P500 (New York), CAC40 (Paris), Singapore All Shares, and the Japanese Nikkei, show that in almost all of the series the unit root hypothesis cannot be rejected. The Hang Seng and the Singapore All Shares seem to be the most nonstationary series with orders of integration higher than one, and the S&P500 is the less nonstationary series, with values smaller than one and showing mean reversion.

JEL classification: C22; G14
Keywords: Stock market; Unit roots; Long memory; Mean reversion

1. Introduction

An important issue in the empirical analysis of financial time series is whether holding-period returns on a risky asset are serially independent, as required by the efficient market hypothesis in its weak form (i.e., the current stock prices fully reflect all the past stock prices information.). Although a precise formulation of an empirically refutable efficient market hypothesis must be model-specific, historically, most such tests focus on the forecastability of common stock returns.

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In this paper I use fractionally integrated techniques to revisit the long memory and mean reversion issues in the stock markets. In particular, I examine if the stock market indexes can be specified in terms of $I(d)$ statistical models, with the possibility that d is a fractional number.

For stock indexes, the evidence in favor of long memory may be due to the effect of aggregation. In fact, aggregation is one of the main sources of long memory. The key idea is that aggregation of independent weakly dependent series can produce a strong dependent series: Granger (1980) and Robinson (1978) show that fractional integration can arise as a result of aggregation when: (a) data are aggregated across heterogeneous autoregressive (AR) processes, and (b) data involving heterogeneous dynamic relationships at the individual level are then aggregated to form the time series. Moreover, the existence of long memory in financial asset returns suggests that new theoretical models based on nonlinear pricing models should be elaborated. Mandelbrot (1971) notes that in the presence of long memory, martingale models of asset prices cannot be obtained from arbitrage. In addition, statistical inference concerning asset pricing models based on standard testing procedures may not be appropriate in the context of long memory processes (see, e.g., Barkoulas, Baum, & Travlos, 2000; Yajima, 1985).

The paper proceeds as follows. In Section 2 I briefly review the literature on modeling stock market indexes using long memory processes. In Section 3 I define the concept on fractional integration. In Section 4 I describe some of the most frequently used techniques for fractional integration, with special emphasis on a parametric and a semiparametric method that have some distinguishing features compared with other methods. In Section 5, I apply these procedures to several stock market indexes. Section 6 concludes.

2. Historical background

Within the paradigm of the efficient market hypothesis, which has been broadly categorized as the "random walk" theory of stock prices, the evidence is mixed. For instance, using a variance-ratio test, Lo and MacKinley (1988) and Poterba and Summers (1988), conclude that stock returns exhibit mean reversion. Fama and French (1988), who examine the autocorrelations of one-period returns, also find mean reversion. By contrast, using a generalized form of rescaled range (R\S) statistic, Lo (1991) finds no evidence against the random walk hypothesis. Using annual data and allowing for fractional alternatives, Caporale and Gil-Alana (2002) report that US stock returns are close to being an $I(0)$ series, and point out that their degree of predictability depends on the process followed by the error term.

As mentioned in the introduction, in this paper I focus on fractional integration and long memory behavior. The literature in this topic has increased in recent years, but the results are mixed. Some authors find little or no evidence of long memory in stock markets (see, e.g., Hiemstra & Jones, 1997, and the references therein). Barkoulas and Baum (1996), Barkoulas et al. (2000), Cheung and Lai (1995), Crato (1994), Henry (2002), Sadique and Silvapulle (2001), and Tolvi (2003) are among those who find evidence of long memory in monthly, weekly, and daily stock market returns.

The study of long range dependence clearly requires a sufficiently long series to justify the application of large sample inference rules based on semiparametric models. However, there is not yet a finite sample theory for rules of parametric inference on long memory.

Several recent papers use the Standard and Poor's (S&P) 500 index of over 17,000 daily observations. Granger and Ding (1995a, 1995b) focus on power transformation or absolute value of the returns (which they use as proxies of volatility). They estimate a long memory process to study persistence in volatility,