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Computing option pricing models under transaction costs☆

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ABSTRACT

This paper deals with the Barles–Soner model arising in the hedging of portfolios for option pricing with transaction costs. This model is based on a correction volatility function Ψ solution of a nonlinear ordinary differential equation. In this paper we obtain relevant properties of the function Ψ which are crucial in the numerical analysis and computing of the underlying nonlinear Black–Scholes equation. Consistency and stability of the proposed numerical method are detailed and illustrative examples are given.

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1. Introduction

In a complete financial market without transaction costs, the Black–Scholes (B–S) no-arbitrage argument provides a rational option pricing formula and a hedging portfolio that replicates the contingent claim. Under the transaction costs, the continuous trading required by the hedging portfolio is prohibitively expensive, [1]. Several alternatives lead to option prices that are equal to Black–Scholes price but with an adjusted volatility. In 1992, Boyle and Vorst [2], derived from a binomial model an option price taking into account transaction costs and that is equal to a B–S price but with a modified volatility of the form

$$\sigma = \sigma_0(1 + cA)^{1/2}, \quad A = \frac{\mu}{\sigma_0\sqrt{\Delta T}}, \; c = 1.$$

Here, μ is the proportional transaction cost, Δt the transaction period, and σ_0 is the original volatility constant. Leland [3] computed $c = (\frac{2}{\pi})^{1/2}$. Kusuoka [4] then showed that the "optimal" c depends on the risk structure of the market. Paras and Avellaneda [5] derived the modified volatility

$$\sigma = \sigma_0(1 + A \operatorname{sign}(V_{SS}))^{1/2},$$

from a binomial model using the algorithm of Bensaid et al. [6]. Whalley and Wilmott [7] using an asymptotic analysis based on [8] propose the same adjusted volatility. A comparison of the exact hedging strategy of [8] and the asymptotic hedging strategy of [7] has been studied in [9]. Here, V is the option price, S the price of the underlying asset, and V_{SS} denotes the second derivative of V with respect to S (the "Gamma"). In particular, the option price does not need to be convex.

Kratka in [10] and Jandačka and Ševčovič in [11] propose a correction of volatility of the form

$$\sigma^2 = \sigma_0^2(1 + \mu(S\, V_{SS})^{\frac{1}{3}}),$$

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where $\mu = 3(C^2R/2\pi)^{\frac{1}{3}}$ and C, R are nonnegative constants representing the transaction cost measure and the risk premium measure, respectively.

A more complex model has been proposed by Barles and Soner [1], assuming that investor's preferences are characterized by an exponential utility function. In their model the nonlinear volatility reads

$$\sigma^2 = \sigma_0^2(1 + \Psi[\exp(r(T-t)a^2S^2V_{SS})]), \tag{1.1}$$

where r is the risk-free interest rate, T the maturity, and $a = \mu\sqrt{\gamma N}$, with risk aversion factor γ and the number N of options to be sold. The function Ψ is the solution of the nonlinear initial value problem

$$\Psi'(A) = \frac{\Psi(A) + 1}{2\sqrt{A\Psi(A)} - A}, \quad A \neq 0,\ \Psi(0) = 0. \tag{1.2}$$

In the mathematical literature, only a few results can be found on the numerical discretization of B–S equation, mainly for linear B–S equations. The numerical approaches vary from finite element discretizations [12,13], finite-difference approximations [14–16]. The numerical discretization of the B–S equations with the nonlinear volatility (1.2) has been performed using explicit finite-difference schemes [1]. However, explicit schemes have the disadvantage that restrictive conditions on the discretization parameters (for instance, the ratio of the time and the space step) are needed in order to obtain stable, convergent schemes [17]. Moreover, the order of convergence is only one in time and two in space. [18] combines high-order compact difference schemes derived by [19] and techniques to construct numerical solutions with frozen values of the nonlinear coefficient of the nonlinear B–S equation to make the formulation linear.

In this paper we use a semidiscretization technique by using fourth-order difference approximations of the partial derivatives V_S and V_{SS} arising in the nonlinear B–S equation

$$V_t + \frac{1}{2}\sigma(V_{SS})^2S^2V_{SS} + rSV_S - rV = 0. \tag{1.3}$$

Then we achieve an ordinary system of nonlinear ordinary differential equations with respect to the time, that is solved numerically. Apart form (1.3), in the Barles–Soner model one has the terminal condition

$$V(S,T) = \max(0, S-E), \quad S > 0, \tag{1.4}$$

and the boundary conditions

$$V(0,t) = 0, \quad \lim_{s\to\infty} \frac{V(S,t)}{S - Ee^{-r(T-t)}} = 1. \tag{1.5}$$

In order to compute the numerical solution, it is necessary to work in a bounded domain. Once this numerical domain has been chosen, the boundary conditions can be translated from the asymptotic condition (1.5), as it is done for instance in [1] or [18], or the boundary values must be found together with the solution and they are linked with the rest of the numerical solution in the interior of the numerical domain by using extrapolation techniques. This last approach is used in this paper in accordance with the used scheme.

Using the change of variable $\tau = T - t,\ U(S,\tau) = V(S,t)$ Eq. (1.3) together with the initial condition (1.4) is transformed into

$$U_\tau - \frac{S^2}{2}\sigma^2U_{SS} - rSU_S + rU = 0, \quad 0 < S < \infty,\ 0 < \tau \le T, \tag{1.6}$$

$$U(S,0) = \max(0, S-E). \tag{1.7}$$

This paper is organized as follows. Section 2 is addressed to the study of the properties of the volatility correction function Ψ after obtaining the implicit solution of (1.2). In Section 3, by using semidiscretization with respect to S one gets a nonlinear system of ordinary differential equations with respect to the time, and then it is discretized using a forward explicit scheme. This approach allows us to study the stability and consistency of the nonlinear scheme in Sections 4 and 5 without using linearization strategies as it is done in [18]. Section 6 includes illustrative examples of European call option pricing where the computed numerical solution and their properties are checked.

If A is a matrix in $\mathbb{R}^{p\times p}$ and A^t denotes its transposed matrix, we denote by $\|A\|$ the spectral norm of A defined as, [20],

$$\|A\| = \max\left\{\sqrt{\lambda};\ \lambda \text{ eigenvalue of } A^tA\right\}.$$

If q is an integer with $|q| \le p-1$, and A_q is a band matrix in $\mathbb{R}^{p\times p}$ such that $A_q = (a_{ij})$ with $a_{ij} = 0$ everywhere outside of the diagonal $j = i + q$, then it is easy to show that

$$\|A_q\| = \max\left\{|a_{i,i+q}|;\ \begin{matrix} 1 \le i \le p-q \text{ if } q \ge 0 \\ 1-q \le i \le p \text{ if } q < 0 \end{matrix}\right\}. \tag{1.8}$$

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