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A Black-Scholes Schrödinger option price: 'bit' versus 'qubit'

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Abstract

The celebrated Black-Scholes differential equation provides for the price of a financial derivative. The uncertainty environment of such option price can be described by the classical 'bit': a system with two possible states. This paper argues for the introduction of a different uncertainty environment characterized by the so called 'qubit'. We obtain an information-based option price and discuss the differences between this option price and the classical option price.

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Keywords: Bit; Qubit; Time scale; Black-Scholes pde

1. Introduction

One of the most celebrated equations in finance is the Black-Scholes differential equation, which draws quite heavily from the physics based notion of Brownian motion. In the finance literature there exists a close connection between the binomial option pricing model and the Black-Scholes model [1]. The Black-Scholes model can be derived from the binomial model. In the latter the stock price can take on two different positions at each time step. In analogy with information theory, the binomial model represents a bit: a system with two possible states (at each time step). Other examples where the bit notion is also implicitly used is in the so called 'Arrow-Debreu' paradigm, where future payments are a function of both time and the states of the world [2,3]. In the words of W.F. Sharpe, a well recognized finance academic, this paradigm is part of what he calls 'nuclear financial economics' [4].

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This paper has as aim to show how the Black-Scholes option price does change when we extend the notion of bit into that of qubit. If the states '0' and '1' are the basis states of the bit then a qubit is a linear combination of those basis states. The paper is organized as follows. In the next section we rationalize, from a finance perspective, the use of the Schrödinger equation. We also consider the value of the option price within the qubit environment and provide for an example. Finally, we consider the existence of the Black-Scholes portfolio in the qubit environment and in the bit environment but for a different time scale.

2. Rationalization and solution of the Schrödinger equation in finance

When extending the bit into a qubit the uncertainty environment changes. If a stock price is subjected to a bit or 0-1 state system then in the next period of time the stock can either move up or down. In the qubit system, the stock price can adopt any combination of the up and down movements in the next period of time. Heuristically, one may therefore claim that the evolution of the stock price in the next period of time is far more uncertain in the qubit system than it is in the bit system. It needs to be remarked however that the total information content of the qubit is still one bit! [5]. We can link uncertainty with lack of information. If each possible move of the stock price in the next instant of time carries information, then each such move carries much less information in the qubit environment than in the bit environment. From a decision maker point of view, what counts is an *informational* assessment, via the use of a probability for instance, of the next price move. This informational assessment will be very difficult to carry out in the qubit environment. Says Chen 'there do not exist probabilities of future up and down movements in the stock price...in the quantum case' [6].

The Planck constant, $\hbar$, will distinguish the transition from a qubit environment ($\hbar > 0$) to a bit environment ($h \to 0$). The Black-Scholes pde takes the following format:

$$\frac{\partial f(S,t)}{\partial t} + rS\,\frac{\partial f(S,t)}{\partial S} + \frac{1}{2}\,\sigma^2 S^2\,\frac{\partial^2 f(S,t)}{\partial S^2} = rf(S,t)\,,$$

where $f(S,t)$ is the option price, r is the risk free rate of interest, t is maturity time, S is the stock price and σ is volatility. The Black-Scholes pde is a heat equation. An important difference between the Schrödinger equation and the heat equation is the factor i, the square root of -1. One can remove this difference by a time variable change. Aside from the closeness of the heat equation to the Schrödinger equation we can further rationalize the use of the latter equation by asserting that it only meaningfully exists for the case where $\hbar > 0$, which is the parameter distinguishing the qubit from the bit environment. The Schrödinger equation is

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(S,t) = -\frac{\hbar^2}{2m}\,\frac{\partial^2}{\partial S^2}\Psi(S,t) + V(S,t)\Psi(S,t)\,,$$