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The time-varying and asymmetric dependence between crude oil spot and futures markets: Evidence from the Mixture copula-based ARJI–GARCH model

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ABSTRACT

This paper designs a Mixture copula-based ARJI–GARCH model to simultaneously investigate the dynamic process of crude oil spot and futures returns and the time-varying and asymmetric dependence between spot and futures returns. The individual behavior of each market is modeled by the ARJI–GARCH process. The time-varying and asymmetric dependence is captured by the Mixture copula which is composed of the Gumbel copula and Clayton copula. Empirical results show three important findings. First, jumping behavior is an important process for each market. Second, spot and futures returns do not have the same jump process. Third, the tail dependence between spot and futures markets is time-varying and asymmetric with the magnitude of upper tail dependence being slightly weaker than that of lower tail dependence.

1. Introduction

A growing body of studies has investigated the impact of crude oil price on economic and financial developments and the findings indicate a negative effect. These reports have stimulated subsequent research that aims to better understand crude oil prices in spot and futures markets. Due to the unusually large changes that have recently occurred in these markets and their asymmetric relationship, the current paper proposes a Mixture copula-based ARJI–GARCH model to investigate both the interdependence of oil spot and futures prices and their individual dynamic behaviors.

Previous studies in this area have provided limited results. Studies show that generalized autoregressive conditional heteroskedasticity (GARCH) model and its extended models are very popular approach in modeling and forecasting spot return and futures return. For exploring the linear relationship between spot and futures returns, the constant correlation coefficient (CCC) GARCH and the dynamic conditional correlation (DCC) GARCH have been adopted.[1] However, the aforementioned approaches are hampered in their ability to successfully capture the true dynamic relationship between spot and futures markets. The reason is that they have yet to provide a flexible and reasonable way to jointly investigate the marginal behavior of these markets and their time-varying and asymmetric relationship.

Recent studies have indicated that the processes of crude oil spot and futures prices heavily rely on many components, including usual factors (e.g., routine energy consumptions and productions) and unusual factors (e.g., unexpected demand and supply shocks). The aforementioned GARCH models are only suitable for modeling the volatility clustering characteristics caused by the usual factors. However, when sudden and large price changes happen, the adequacy of the GARCH models patently cannot be guaranteed. To overcome this defect, Chan and Maheu (2002) propose the autoregressive conditional jump intensity (ARJI) model, which simultaneously considers the persistence in conditional variance and irregular price changes. This model seems to be a better specification in describing the crude oil price dynamics. For example, Lee et al. (2010) utilize the ARJI specification to investigate the time series properties of the West Texas Intermediate crude oil spot and futures markets and demonstrate that periods of sudden large changes in crude oil price can be appropriately identified through the jump intensity, which describes the possible number of jumps at each time point.[2]

Several empirical studies have documented the importance of the fat-tailed property in accurately forecasting the Value-at-Risk (VaR) of crude oil spot and futures prices. For example, Giot and Laurent

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[1] There are also many different bivariate GARCH models. See Bauwens et al. (2006) for details.

[2] The ARJI–GARCH model has been widely used to investigate the behaviors of financial assets. For example, Chan and Maheu (2002), Maheu and McCurdy (2004), Jondeau and Rockinger (2006), Daal et al. (2007) and Chiou and Lee (2009) use this model to analyze the stock prices; Chen and Shen (2004) use the mode to explore the exchange rate market; and Das (2002) uses it to discuss the process of interest rates.

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(2003) demonstrate that the skewed student distribution is better in evaluating VaR forecasts than the normal distribution. Fan et al. (2008) find that the generalized error distribution (GED) has better performance in forecasting VaR than the normal distribution. On the other hand, Agnolucci (2009) shows that the accuracy of volatility forecasts hinges on the distribution specification and that the GED specification performs better at forecasting conditional variance than the normal specification. With respect to the ARJI–GARCH model, an advantage worth mentioning is that it can generate skewed and fat-tailed conditional density even if the assumption that error terms are normally distributed holds.[3]

Although the correlation coefficient, determined by the bivariate GARCH model with the elliptic distributed error terms (e.g., the normal distribution or Student's t distribution), is the most popular measure for describing the linear relationship between crude oil spot and futures returns, it cannot capture their nonlinear relation. This is a critical drawback, since the nonlinear relation between spot and futures returns has become an important issue in recent years. Another drawback is that the correlation measure may misestimate their linear relationship due to the incorrect specification of the bivariate distribution. The copula approach developed by Patton (2006) can remedy the above two weaknesses simultaneously. It can deal with the computational difficulty in estimating complex joint distribution, and measure the nonlinear interdependence through two measures: the tail dependence index and Kendall's tau index. The main reason for the popularity of the copula approach is that the Sklar (1959) theorem indicates that the joint density of two returns can be obtained if both the marginal density of each return and the copula function can be correctly specified.

It is well-known that the high correlation coefficient between spot and futures markets could be the reason that the futures product is a good hedge instrument for the corresponding spot good: futures prices contain information that can help to forecast spot prices. However, this high linkage does not mean that the two markets, in most cases, move in the same direction, irrespective of an increase or decrease in price. Moreover, the high linkage does not guarantee that the two markets will have a symmetric correlation. Unlike the existing studies which use the correlation coefficient to measure the linkage between crude oil spot and futures markets, this paper focuses on the possible asymmetric tail dependence and the possible asymmetric comovement.

Therefore, this paper develops a Mixture copula-based ARJI–GARCH model to discuss the nonlinear dependence between crude oil spot and futures markets. In essence, combining the ARJI–GARCH model with the mixture copula specification yields a new model. The main advantages of this new proposed model are briefly introduced as follows. First, in order to take the vital characteristics (e.g., volatility clustering, a sudden large variation in price, cointegration and the non-normal distribution) found in the extant literature into account, the ARJI–GARCH model with the long-term disequilibrium adjustment term (the so-called error correction term) is used to model the process for crude oil spot and futures returns.[4] Moreover, to investigate whether they are affected by the same factors, the research allows the spot and futures returns to have different jump processes.

Second, the mixed copula is composed of the Gumbel copula, which focuses only on the right-tail structure, and the Clayton copula, which concentrates attention only on the left-tail structure. Hence, the Mixture copula allows the asymmetric dependence, i.e., the comovement structure between spot and futures returns in the extreme right tail, to be different from that in the extreme left tail. Moreover, the nonlinear dependence is allowed to be time-varying. In comparison to the Gaussian copula, Student's t copula, Gumbel copula and Clayton copula,[5] the Mixture copula has more flexibility in describing the joint dependence over time without assuming complex joint distributions.

The remainder of this paper is organized as follows. Section 2 briefly introduces the copula theory and then describes the marginal distributions with the ARJI–GARCH process and four copula densities. Section 3 reports the empirical results and discusses the asymmetric dependency relation. Section 4 provides the conclusions.

2. The Mixture copula-based ARJI–GARCH model

This section introduces the basic principle of the copula theorem. According to the work of Patton (2006), the relationship between the conditional copula function and the joint conditional distribution function can be expressed as:

$$F\left(r_{s,t}, r_{f,t}\middle|\Omega_{t-1}\right) = C\left(u_{s,t}, u_{f,t}\middle|\Omega_{t-1}\right) \tag{1}$$

where $r_{s,t}$ is the spot return, $r_{f,t}$ is the futures return, F denotes the joint cumulative distribution, Ω_{t-1} refers to the information set at time $t-1$, C is the copula function, $u_{s,t}$ denotes the cumulative distribution value of spot return, and $u_{f,t}$ denotes the cumulative distribution value of futures return.

Based on the viewpoint of density function, the above equation can be rewritten as:

$$f\left(r_{s,t}, r_{f,t}\middle|\Omega_{t-1}\right) = \left\{f_s\left(r_{s,t}|\Omega_{t-1}\right) \times f_f\left(r_{f,t}\middle|\Omega_{t-1}\right)\right\} \times c\left(u_{s,t}, u_{f,t}\middle|\Omega_{t-1}\right) \tag{2}$$

where f refers to the probability density function and c is the copula density. Moreover, the log-likelihood function can be defined as

$$\begin{aligned}\ln L = &\left\{\sum_{t=1}^{T}\left(\ln f_s\left(r_{s,t}|\Omega_{t-1}\right) + \ln f_f\left(r_{f,t}\middle|\Omega_{t-1}\right)\right)\right\}\\ &+\sum_{t=1}^{T}\ln c\left(u_{s,t}, u_{f,t}\middle|\Omega_{t-1}\right)\end{aligned} \tag{3}$$

It is clear from Eq. (3) that the log-likelihood function is composed of two parts: the one is constructed by two marginal density functions and the other is distinguished by the copula density. The IFM (inference for margins) method suggested by Patton (2006) is used to obtain the maximum likelihood estimates. It is a two-step estimation procedure. The first step is to estimate parameters specified in the marginal density of each return. The second step is to estimate parameters of copula density based on the estimation results from the first step. The marginal specification and the copula structure are introduced in the next two subsections.

2.1. Marginal density specification

This subsection introduces the behavior dynamics for spot and futures returns. In order to take into account the price characteristics mentioned in the Introduction, the research extends the ARJI–GARCH model of Chan and Maheu (2002) as follows:

$$r_{i,t} = c_i + \sum_{m=1}^{bm} a_{im} r_{i,t-m} + \sum_{n=1}^{bn} a_{kn} r_{k,t-n} + a_{iec} EC_{t-1} + \varepsilon_{1,t}^{i} + \varepsilon_{2,t}^{i}, \tag{4}$$

[3] The detailed formulas for the first four conditional moments of the ARJI model can be obtained from the work of Maheu and McCurdy (2004). The conditional skewness and kurtosis coefficients are related to the mean and variance of jump size as well as the jump intensity.

[4] In studying the relationship between crude oil spot and futures markets, Alizadeh et al. (2008), Lien and Yang (2008) and Huang et al. (2009) demonstrate that the error correction term is an important explanatory variable.

[5] The Gaussian copula has no tail dependence. The Student's t copula has symmetric tail dependence. The Gumbel and Clayton copulas have a right-tail dependence and left-tail dependence, respectively. See Section 2 for an introduction to the copula specifications.