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Power tails of index distributions in chinese stock market

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Abstract

The power α of the Lévy tails of stock market fluctuations discovered in recent years are generally believed to be universal. We show that for the Chinese stock market this is not true, the powers depending strongly on anomalous daily index changes short before market closure, and weakly on the opening data.

Keywords: Power-law; Lévy distribution; Stock market index

The study of price fluctuations of financial assets has long been an active field of finance [1–5]. In recent years, the possibility to access and analyze huge amounts of high-frequency financial data with the help of contemporary computer technology has attracted the interest of theoretical physicists [6–13], and led to several significant empirical findings [14,15,18–21]. One of these is that the distribution of stock price fluctuations [22] is characterized by a power-law decay with an exponent $\alpha \simeq 3$, which lies well outside the Lévy-stable range $0<\alpha<2$ [23–27]. This is the so-called *inverse cubic law* of returns. The observed tails are absent in the previous widely used distributions [12], most prominently in the normal distribution proposed by Bachelier [1], which forms the basis of the Black–Scholes theory, in the truncated Lévy distribution of Mantegna and Stanley [15], in the Meixner distributions [16], or in the generalized hyperbolic distributions [17]. Although the tails can be fitted with the pure Lévy distribution as proposed by Mandelbrot [5], the distribution of the central most probably events cannot.

Remarkably the empirical tail behavior appears to be *universal*, because it holds for stocks of different economies, such as German stocks [23], US stocks [24,26], as well as various market indices such as the S&P 500, the Dow Jones, the NIKKEI, the Hang Seng, the Milan, and the DAX index [23,25,27]. In addition, the scaling behavior of the distribution has been analyzed for US stocks and market indices [25,26], which indicates that the tails of the distribution are well described by a power-law decay for time scales Δt from 1 min up to a certain value $(\Delta t)_x$ [28]. By the central limiting theorem the distributions converge, of course, to

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Gaussian behavior for Δt longer than $(\Delta t)_x$, albeit quite slowly. All these empirical findings suggest that there might be some universal underlying regularities governing the complex financial systems [29–32].

In order to explain the mechanism of the empirical power-law distribution, several theoretical models have been proposed [33–35]. For instance, Solomon and Richmond [33] build a multiagent system by the use of a generalized Lotka-Volterra model. A theory of Gabaix et al. [34] find this behavior on the basis of the economic optimization by heterogeneous agents.

Such empirical results and corresponding theories have suggested the universality of the power-law distribution of stock price fluctuations. The origin of this, however, is far from understood. When Huang [36] analyzed the 1-min data of the Hang Seng index in the Hang Kong stock market from January 1994 to December 1997, he found that the tail properties of the probability distribution of index fluctuations depend on the opening effect of each trading day. When skipping the data in the first 20 min of each morning session, the asymptotic behavior of the probability distribution shows an exponential-type decay as $P(x) \sim \exp(-\alpha|x|)/|x|$, where the index move $x = x(t)$ over a time scale Δt satisfies $x(t) = \text{index}(t) - \text{index}(t - \Delta t)$. Moreover, he claims that this empirical result can be derived from a Langevin approach [37]. In contrast, the case without any skip of the data is characterized by a power-law decay with an exponent $\alpha \simeq 3$.

Further doubts on the universality were raised by K. Matia et al. [38], who tested the daily returns from November 1994 to June 2002 for the 49 largest stocks of the National Stock Exchange, which has the highest volume of trade in India. These authors found an exponential probability density function of normalized returns g, to be defined below, as $P(g) \sim \exp(-\beta g)$, with the characteristic decay scales $\beta = 1.51 \pm 0.05$ for the negative tail and $\beta = 1.34 \pm 0.04$ for the positive tail. This led them to suggest that the power-law behavior merely holds for highly developed economies while the less highly developed ones follow a scale-dependent behavior.

On the other hand, most financial markets exhibit rich patterns caused by periodic market closures. For example, Cajueiro et al. [39] found that the intensity of the long-range dependence phenomena presented in this market depends on the time of the day that the phenomena is measured. This kind of pattern seems to be related to trading performed by different types of investors and the flow of information over the day.

The purpose of this paper is to test this hypothesis by analyzing the distributions of the Chinese market, which should be comparable to the Indian one. In particular, we consider the influence of the opening effect, and of the near-closure returns [40]. In addition, we study the scaling behavior of the distribution for time scales from 1 min up to 64 min.

We analyze the Shanghai Stock Exchange (SSE) index [41], which is a market-value weighted index. The data contain minute-by-minute records of every trading day from November 2000 to June 2004 [42], with the total number of data exceeding 2×10^5.

Define $S(t)$ as the value of SSE index and the return $G(t)$ over a time scale Δt as the forward change in the logarithm of $S(t)$, namely, $G(t) = \ln S(t + \Delta t) - S(t)$. Then we define the normalized return $g(t)$ as,

$$g(t) = \frac{G(t) - \langle G(t) \rangle_T}{\sigma}. \tag{1}$$

Here σ is the standard deviation of $G(t)$, and $\langle \cdots \rangle_T$ denotes an average over the entire length of the time series.

Fig. 1(a) and (b) display the cumulative distributions of 1-min returns of the SSE index for the positive and the negative tails, respectively. When the distributions are calculated with all data, which include near-closure returns, both positive and negative tails cannot be fitted well by the regression fits, especially for large values of g [43]. Consequently, in this case we obtain $\alpha \simeq 2.46$ in the region $1 \leqslant g \leqslant 8$, and $\alpha \simeq 1.52$ in the region $8 \leqslant g \leqslant 20$ for the positive tail. Likewise, we obtain $\alpha \simeq 2.64$ in the region $1 \leqslant g \leqslant 4$, and $\alpha \simeq 1.72$ in the region $4 \leqslant g \leqslant 20$ for the negative tail (see Eq. (2) for the definition of α). According to these results, there is definite evidence for a power-law asymptotic behavior described by Lévy distributions with $\alpha \approx 3$.

When the irregular near-closure returns are removed we still observe, for both positive and negative tails, a power-law asymptotic behavior

$$P(g > x) \sim x^{-\alpha}. \tag{2}$$

The powers of these distributions are completely consistent with the previous empirical power-law distributions found for different stock markets [23–27].