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An optimal dividends problem with transaction costs for spectrally negative Lévy processes

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ABSTRACT

We consider an optimal dividends problem with transaction costs where the reserves are modeled by a spectrally negative Lévy process. We make the connection with the classical de Finetti problem and show in particular that when the Lévy measure has a log-convex density, then an optimal strategy is given by paying out a dividend in such a way that the reserves are reduced to a certain level c_1 whenever they are above another level c_2. Further we describe a method to numerically find the optimal values of c_1 and c_2.

1. Introduction

In this paper we consider an offshoot of the classical de Finetti's optimal dividends problem in continuous time for which a transaction cost is incurred each time a dividend payment is made. Because of this fixed cost, it is no longer feasible to pay out dividends at a certain rate and therefore only lump sum dividend payments are possible.

Within this problem we assume that the underlying dynamics of the risk process is described by a spectrally negative Lévy process which is now widely accepted and used as a replacement for the classical Cramér–Lundberg process (cf. Albrecher et al. (2008), Avram et al. (2007), Dufresne and Gerber (1993), Dufresne et al. (1991), Furrer (1998), Huzak et al. (2004), Kyprianou and Palmowski (2007), Kyprianou et al. (in press), Loeffen (2008, 2009) and Renaud and Zhou (2007)). Recall that a Cramér–Lundberg risk process $\{X_t : t \geq 0\}$ corresponds to

$$X_t = x + ct - \sum_{i=1}^{N_t} C_i, \tag{1}$$

where $x > 0$ denotes the initial surplus, the claims $C_1, C_2, \ldots$ are i.i.d. positive random variables with expected value μ, $c > 0$ represents the premium rate and $N = \{N_t : t \geq 0\}$ is an independent Poisson process with arrival rate λ. Traditionally it is assumed in the Cramér–Lundberg model that the net profit condition $c > \lambda\mu$ holds, or equivalently that X drifts to infinity. In this paper X will be a general spectrally negative Lévy process and the condition that X drifts to infinity will not be assumed.

We will now state the control problem considered in this paper. As mentioned before, $X = \{X_t : t \geq 0\}$ is a spectrally negative Lévy process which is defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t : t \geq 0\}, \mathbb{P})$ satisfying the usual conditions. Within the definition of a spectrally negative Lévy process it is implicitly assumed that X does not have monotone paths. We denote by $\{\mathbb{P}_x, x \in \mathbb{R}\}$ the family of probability measures corresponding to a translation of X such that $X_0 = x$, where we write $\mathbb{P} = \mathbb{P}_0$. Further $\mathbb{E}_x$ denotes the expectation with respect to $\mathbb{P}_x$ with $\mathbb{E}$ being used in the obvious way. The Lévy triplet of X is given by (γ, σ, ν), where $\gamma \in \mathbb{R}$, $\sigma \geq 0$ and ν is a measure on $(0, \infty)$ satisfying

$$\int_{(0,\infty)} \left(1 \wedge x^2\right) \nu(\mathrm{d}x) < \infty.$$

Note that even though X only has negative jumps, for convenience we choose the Lévy measure to have only mass on the positive instead of the negative half line. The Laplace exponent of X is given by

$$\psi(\theta) = \log\left(\mathbb{E}\left(\mathrm{e}^{\theta X_1}\right)\right) = \gamma\theta + \frac{1}{2}\sigma^2\theta^2$$

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$$-\int_{(0,\infty)}\left(1-e^{-\theta x}-\theta x\mathbf{1}_{\{0<x<1\}}\right)\nu(dx)$$

and is well defined for $\theta \geq 0$. Note that the Cramér–Lundberg process corresponds to the case that $\sigma = 0$, $\nu(dx) = \lambda F(dx)$ where F is the law of C_1 and $\gamma = c - \int_{(0,1)} x\nu(dx)$. The process X will represent the risk process/reserves of the company before dividends are deducted.

We denote a dividend or control strategy by π, where $\pi = \{L_t^\pi : t \geq 0\}$ is a non-decreasing, left-continuous $\mathbb{F}$-adapted process which starts at zero. Further we assume that the process L^π is a pure jump process, i.e.

$$L_t^\pi = \sum_{0\leq s<t} \Delta L_s^\pi \quad \text{for all } t \geq 0. \tag{2}$$

Here we mean by $\Delta L_s^\pi = L_{s+}^\pi - L_s^\pi$ the jump of the process L^π at time s.

The random variable L_t^π will represent the cumulative dividends the company has paid out until time t under the control π. We define the controlled (net) risk process $U^\pi = \{U_t^\pi : t \geq 0\}$ by $U_t^\pi = X_t - L_t^\pi$. Let $\sigma^\pi = \inf\{t > 0 : U_t^\pi < 0\}$ be the ruin time and define the value function of a dividend strategy π by

$$v_\pi(x) = \mathbb{E}_x\left[\int_0^{\sigma^\pi} e^{-qt} d\left(L_t^\pi - \sum_{0\leq s<t}\beta\mathbf{1}_{\{\Delta L_s^\pi>0\}}\right)\right],$$

where $q > 0$ is the discount rate and $\beta > 0$ is the transaction cost incurred for each dividend payment. Note that because of (2) we can write $v_\pi(x) = \mathbb{E}_x\left[\sum_{0\leq t<\sigma^\pi} e^{-qt}(\Delta L_t^\pi - \beta\mathbf{1}_{\{\Delta L_t^\pi>0\}})\right]$. By definition $v_\pi(x) = 0$ for $x < 0$. A strategy π is called admissible if ruin does not occur due to a lump sum dividend payment, i.e. $\Delta L_t^\pi \leq U_t^\pi$ for $t < \sigma^\pi$. Let Π be the set of all admissible dividend policies. The control problem consists of finding the optimal value function v_* given by

$$v_*(x) = \sup_{\pi\in\Pi} v_\pi(x)$$

and an optimal strategy $\pi_* \in \Pi$ such that

$$v_{\pi_*}(x) = v_*(x) \quad \text{for all } x \geq 0.$$

Since control strategies of the form (2) are known as impulse controls, we refer to this problem as the impulse control problem.

An important type of strategy for the impulse control problem is the one we call in this paper the $(c_1; c_2)$ policy and which is similar to the well known (s, S) policy appearing in inventory control models, see e.g. Bather (1966) and Sulem (1986). The $(c_1; c_2)$ policy is the strategy where each time the reserves are above a certain level c_2, a dividend payment is made which brings the reserves down to another level c_1 and where no dividends are paid out when the reserves are below c_2. In case X is a Brownian motion plus drift, Jeanblanc-Picqué and Shiryaev (1995) showed that an optimal strategy for the impulse control problem is formed by a $(c_1; c_2)$ policy. Paulsen (2007) considered the case when X is modeled by a diffusion process and showed that under certain conditions a $(c_1; c_2)$ policy is optimal. Note that in Paulsen (2007) this type of strategy is referred to as a lump sum dividend barrier strategy. Further, Alvarez and Rakkolainen (2009) study the case where the driving process is a spectrally negative Lévy diffusion with a jump component of geometric form. In this paper we will investigate when an optimal strategy for our impulse control problem is formed by a $(c_1; c_2)$ policy.

When the assumption (2) is dropped and the transaction cost β is taken to be equal to zero, then the impulse control problem transforms into the classical de Finetti optimal dividends problem. The latter optimal dividends problem will be referred to as the de Finetti problem in the remainder of the paper. This particular problem was introduced by de Finetti (1957) in a discrete time setting for the case that the risk process evolves as a simple random walk. Thereafter the de Finetti problem has been studied in a continuous time setting for the case that X is a Cramér–Lundberg risk process (Gerber, 1969; Azcue and Muler, 2005) and for the case that the risk process is a general spectrally negative Lévy process (Avram et al., 2007; Kyprianou et al., in press; Loeffen, 2008). For this problem an important strategy is the so-called barrier strategy. The barrier strategy at level a is the strategy where initially (in case the starting value of the reserves are above a) a lump sum dividend payment is made to bring the reserves back to level a and thereafter each time the reserves reach the level a, non-lump sum dividend payments are made in such a way that the reserves do not exceed the level a, but where no dividends are paid out when the reserves are strictly below a. Mathematically this corresponds to reflecting the risk process X at a. The barrier strategy at level a may be seen (at least intuitively) as a limit of $(c_1; c_2)$ policies where c_1 and c_2 converge to the barrier a.

Gerber (1969) proved that an optimal strategy for the de Finetti problem is formed by a barrier strategy in the case where X is a Cramér–Lundberg risk process with exponentially distributed claims. Building on the work of Avram et al. (2007), Loeffen (2008) showed that optimality of the barrier strategy for the de Finetti problem depends on the shape of the so-called scale function of a spectrally negative Lévy process. To be more specific, the q-scale function of X, $W^{(q)} : \mathbb{R} \to [0,\infty)$ where $q \geq 0$, is the unique function such that $W^{(q)}(x) = 0$ for $x < 0$ and on $[0,\infty)$ is a strictly increasing and continuous function characterized by its Laplace transform which is given by

$$\int_0^\infty e^{-\theta x} W^{(q)}(x) dx = \frac{1}{\psi(\theta) - q} \quad \text{for } \theta > \Phi(q), \tag{3}$$

where $\Phi(q) = \sup\{\theta \geq 0 : \psi(\theta) = q\}$ is the right-inverse of ψ. Theorem 2 of Loeffen (2008) then says that if $W^{(q)}$ is sufficiently smooth and if $W^{(q)\prime}$ is increasing on (a^*, ∞) where a^* is the largest point where $W^{(q)\prime}$ attains its global minimum, then the barrier strategy at a^* is optimal for the de Finetti problem. Here $W^{(q)}$ being sufficiently smooth means that $W^{(q)}$ is once/twice continuously differentiable when X is of bounded/unbounded variation. It was then shown in Loeffen (2008) that when X has a Lévy measure which has a completely monotone density, these conditions on the scale function are satisfied and in particular that $W^{(q)\prime}$ is strictly convex on $(0,\infty)$. (Note that it was shown in Loeffen (2009) that $W^{(q)\prime}$ is actually strictly log-convex.) Shortly thereafter, Kyprianou et al. (in press) proved that $W^{(q)\prime}$ is strictly convex on (a^*, ∞) under the weaker condition that the Lévy measure has a density which is log-convex and then used Theorem 2 from Loeffen (2008) mentioned above, to conclude that the barrier strategy at a^* is optimal (though they needed to relax the sufficiently smoothness assumption). It is important to note that without a condition on the Lévy measure the barrier strategy is not optimal in general. Indeed Azcue and Muler (2005) have given an example for which no barrier strategy is optimal.

In this paper we will show that the results for the de Finetti problem mentioned in the previous paragraph have their counterparts for the impulse control problem, whereby the role of the barrier strategy is now played by the $(c_1; c_2)$ policy. In particular we will give a theorem similar to Theorem 2 in Loeffen (2008) and then use this theorem to show that a certain $(c_1; c_2)$ policy is optimal if the Lévy measure has a log-convex density. Moreover we give an example for which no $(c_1; c_2)$ policy is optimal.

The outline of this paper is as follows. In the next section we review some properties concerning scale functions and in Section 3 we give sufficient conditions under which the $(c_1; c_2)$ policy is optimal. We treat the case when the Lévy measure has a log-convex density in Section 4 and show that the optimal strategy is formed by a unique $(c_1; c_2)$ policy. Further we show how to numerically find the optimal values of c_1 and c_2. In the last section we treat two explicit examples including one for which we show that no $(c_1; c_2)$ policy is optimal.