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Power law and multiscaling properties of the Chinese stock market

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ABSTRACT

We investigate the cumulative probability density function (PDF) and the multiscaling properties of the returns in the Chinese stock market. By using returns data adjusted for thin trading, we find that the distribution has power-law tails at shorter microscopic timescales or lags. However, the distribution follows an exponential law for longer timescales. Furthermore, we investigate the long-range correlation and multifractality of the returns in the Chinese stock market by the DFA and MFDFA methods. We find that all the scaling exponents are between 0.5 and 1 by DFA method, which exhibits the long-range power-law correlations in the Chinese stock market. Moreover, we find, by MFDFA method, that the generalized Hurst exponents $h(q)$ are not constants, which shows the multifractality in the Chinese stock market. We also find that the correlation of Shenzhen stock market is stronger than that of Shanghai stock market.

1. Introduction

It is now generally believed that some dynamic behaviors of an economic system resemble those of a complex system which has been extensively studied in statistical physics. Recent years have seen a large body of literature applying concepts and methods from statistical physics to economics and finance (e.g. Refs. [1,2] among others). Investigating the dynamic behavior of stock markets at the macro level has been an attractive topic in the literature. The key reason for this is that the data of stock-market indices during a relatively long time span have been well documented all around the world. This paper contributes to reveal the non-linear and distribution tail attributes of a particular stock market, Chinese stock market, which represents a relatively young and evolving stock market of a large emerging economy. We find that the return distribution of Chinese stock market follows the power law in the tails, if the timescale is short enough. However, when the timescale is relatively large, say, a day or a week, the tails of the return distribution exhibit the exponential law. We also reveal that the non-linear attribute of Chinese stock market can be characterized by a scaling exponent between 0.5 and 1.

Typical methods used in studying the dynamic behaviors of stock markets include probability distribution functions [3–6], correlation functions [7,8], and network analysis [9]. Among them, the distribution function approach is one of the most intensively studied methods. A wide variety of distributions, such as Pareto–Levy [10,11], Student *t* distribution [12] and stretched exponential distribution [13], has been used in the literature. It has been observed that the power law applies in the return distribution of many stock markets and the exponents varying from 2 to 4 [14,3–6]. In some cases, the power law is also manifest in the trading volumes and volatilities. Furthermore, student t and stretched exponential distributions are proved to be well modeled in some ranges [15–17].

Among various methods to measure correlation and fractality, DFA (Detrended Fluctuation Analysis) has been argued to have many advantages over others [18]. Building on DFA, Kantel and Hardt [19] give an improved version, MFDFA (Multifractal Detrended Fluctuation Analysis), which is capable of studying the multipoint correlation of the non-stable

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series. This method has now been accepted as one of the standard methods to measure multifractality, and has extensively been used since it came into being. Based on the MFDFA and modified R/S analysis, Gu and Zhou [20] study dynamical behavior of the Chinese stock market by investigating the statistical properties. Jiang and Zhou [21] apply the partition function approach and find that the Chinese stock market also exhibited multifractal behavior as a whole. Du and Ning [22] use the partition function approach, singular spectrum method and the MFDFA method to analyze the multifractality in Shanghai stock market and find the weak multifractal features and the long-range correlation. Moreover, the multifractal theories have further been incorporated into the financial risk management framework and some new risk measurement and prediction models have been derived [23–25].

The plan of this paper is as follows. Section 2 presents and analyzes the thin trading. Section 3 uses cumulative PDFs to analyze the return distribution. Section 4 investigates fractality and long-range correlation by DFA method. Section 5 presents multifractal analysis by MFDFA method, while conclusions are presented in Section 6.

2. The thin trading and return calculation

The indices we analyze in this paper are Shanghai stock 50 Index and Shenzhen stock 100 Index. The time span covers from February 2, 1998 through February 27, 2008. We count the time only in trading hours and remove closing hours, weekends, and holidays from the data. Let $p(t)$, $(t = 1, 2, \ldots, n)$, is the Index at time t, and t_1 is the scaling return time, which could be 5 min, 60 min, a day or 1 week, in this paper. We define the logarithmic return as $r(t) = \log p(t) - \log p(t - t_1)$.

It is well known that thin trading can potentially lead to serial correlation in the return series. Thin trading may be the outcome of nonsynchronous trading when stocks trade at every consecutive interval, but not necessarily at the close of each interval. Thin trading can also result from non-trading when stocks do not trade at every consecutive interval. Miller et al. [26] model the effects of both types of thin trading of index portfolio stocks on the observed changes in the index level. This model has been cited extensively, for example, Refs. [27–29]. In particular, they show that an AR(1) model of the following form can capture thin trading for a stock or index:

$$S_t^0 = \partial S_{t-1}^0 + (1 - \partial) S_t \quad (1)$$

where S_t is the true index level innovation, S_t^0 is the observed index level change, and the parameter ∂ measures the degree of trading infrequency. Applying this model to index returns, Miller and his co-authors show that the returns follow a moving average process: the number of moving average components is equal to the number of non-trading days. However, by using the property that the weights decline geometrically with respect to the order of the lag and sum to ∂, they show that the observed index price change process can be written as a modified AR(1) process and the residuals from such a model can be used to adjust the return calculation. Specifically, the following equation is estimated:

$$R_t = \alpha_0 + \partial R_{t-1} + e_t \quad (2)$$

where R_t is the return at time t, α_0 is a constant independent of time t. The residuals from the above equation are used to estimate adjusted returns as follows:

$$R_t^{adj} = e_t / (1 - \partial) \quad (3)$$

where R_t^{adj} is the return at time t adjusted for thin trading.

To account for variation in the adjustment over time, Eq. (2) is estimated recursively to obtain the residuals used to calculate the adjusted returns in Eq. (3). In all the empirical tests of efficiency that follows, we use log returns adjusted for thin trading.

3. Cumulative probability density function of the Chinese stock market

3.1. Data calculation and analysis

It is well documented in the literature that the cumulative probability distribution function (PDF) of the return $r(t)$ has a fat tail [3–6]. The tail of the PDF obeys a power law $P(r) \sim r^{-(1+a)}$, where $P(r)$ is the probability distribution function of $r(t)$. In the double logarithmic coordinates, the cumulative PDFs of Shanghai stock 50 return series with $t_1 = 5$ min, 60 min, 1 day and 1 week are shown in Fig. 1.

The cumulative PDFs of Shenzhen stock 100 return series with $t_1 = 5$ min, 60 min, 1 day and 1 week are shown in Fig. 2.

From Figs. 1 and 2, we observe the power-law distribution at the fat tail. Using the least-square fit, we obtain the power-law exponents, shown in Table 1.

By the analysis of power-law exponents, we find:

(i) The exponents are approximately equal to each other when $t_1 = 5$ min, 60 min or 1 day. This shows the attribute of scale invariance. The exponents of the week timescale series are much larger. This actually indicates the scaling behavior. In other words, the distribution changes when the timescale is as long as a week.
(ii) The exponents of Shanghai 50 Index are generally smaller than those of Shenzhen 100 Index. However, the exponents of Shanghai Index are similar with those of the United States [2].
(iii) Our results are compatible with those in Refs. [22,20,30]. The exponents vary from 2 to 4 on the Chinese stock markets.