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Journal of Economic Dynamics & Control 30 (2006) 1–25
JOURNAL OF Economic Dynamics & Control
www.elsevier.com/locate/econbase

European option pricing and hedging with both fixed and proportional transaction costs

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Received 21 November 2003; accepted 16 November 2004
Available online 23 December 2004

Abstract

In this paper we provide a systematic treatment of the utility based option pricing and hedging approach in markets with both fixed and proportional transaction costs: we extend the framework developed by Davis et al. (SIAM J. Control Optim., 31 (1993) 470) and formulate the option pricing and hedging problem. We propose and implement a numerical procedure for computing option prices and corresponding optimal hedging strategies. We present a careful analysis of the optimal hedging strategy and elaborate on important differences between the exact hedging strategy and the asymptotic hedging strategy of Whalley and Wilmott (RISK 7 (1994) 82). We provide a simulation analysis in order to compare the performance of the utility based hedging strategy against the asymptotic strategy and some other common strategies.

JEL classification: C61; G11; G13

Keywords: Option pricing; Option hedging; Transaction costs; Stochastic impulse control; Markov chain approximation

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0165-1889/$ - see front matter
doi:10.1016/j.jedc.2004.11.002

1. Introduction

The break-through in option valuation theory started with the publication of two seminal papers by Black and Scholes (1973) and Merton (1973). In both papers the authors introduced a continuous time model of a complete friction-free market where the price of a stock follows a geometric Brownian motion. They presented a self-financing, dynamic trading strategy consisting of a riskless security and a risky stock, which replicates the payoff of an option. Then they argued that the absence of arbitrage dictates that the option price be equal to the cost of setting up the replicating portfolio.

In the presence of transaction costs in capital markets the absence of arbitrage argument is no longer valid, since perfect hedging is impossible. Due to the infinite variation of the geometric Brownian motion, the continuous replication policy incurs an infinite amount of transaction costs over any trading interval no matter how small it might be. A variety of approaches have been suggested to deal with the problem of option pricing and hedging with transaction costs. A great deal of them are concerned with the 'financial engineering' problem of either replicating or super-replicating the option payoff. These approaches are mainly preference-free models where rehedging occurs at some discrete time intervals whether or not it is optimal in any sense. However, common sense tells us that an 'optimal' hedging policy should achieve the best possible tradeoff between the risk and the costs of replication. Recognizing the fact that risk preferences differ among individuals, the following conclusion becomes obvious: in pricing and hedging options one must consider the investor's attitude towards risk.

In modern finance it is customary to describe risk preferences by a utility function. Expected utility theory maintains that individuals behave as if they were maximizing the expectation of some utility function of the possible outcomes. Hodges and Neuberger (1989) pioneered the option pricing and hedging approach based on this theory. The key idea behind the utility based approach is the indifference argument: the writing price of an option is defined as the amount of money that makes the investor indifferent, in terms of expected utility, between trading in the market with and without writing the option. In many respects such an option price is determined in a similar manner to a certainty equivalent within the expected utility framework, which is a well grounded pricing principle in economics. The difference in the two trading strategies, with and without the option, is interpreted as 'hedging' the option.

The utility based approach proved to be probably the most successful approach to option hedging with transaction costs. Using simulation analysis, Mohamed (1994), Clewlow and Hodges (1997), and Martellini and Priaulet (2002) demonstrated that the utility based approach achieves excellent empirical performance judging against the best possible tradeoff between the risk and the costs of a hedging strategy.

Hodges and Neuberger (1989) introduced the approach with a fairly general transaction costs structure. However, they carried out computations of the optimal hedging strategies and option prices in a market with only proportional transaction costs, without really presenting the continuous time model and the numerical procedure. Davis et al. (1993) rigorously developed the model of Hodges and