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Economic growth and multiple equilibria: A critical note

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ABSTRACT

Recent literature on economic growth points towards the possibility of the existence of multiple equilibria, under certain conditions. In this paper we use a growth model with a public health infrastructure to analyze, in detail, its local dynamics. The discussion of the existence of equilibria in the model reveals that some candidates for equilibria turn out to be non-relevant and are therefore ruled out. Indeed, multiple equilibria may only arise under restrictive parameter values, meaning that the so called "growth miracles" in some literature may be less likely to occur. Numerical computations illustrate that each relevant equilibrium exhibits local saddle-path stability.

1. Introduction

Literature on economic growth has thoroughly studied the evolution of economies in the long run. From the analysis of transition dynamics, the concept of long-run equilibrium naturally emerges. The Balanced Growth Path (BGP) is a concept of particular importance, as it corresponds to a situation where all relevant economic variables grow at the same rate in the steady state. Hence, one of the main objectives of the subject is to study how economies converge to or diverge from the BGP equilibria and to their local dynamic properties.

A number of authors have recently examined the properties of equilibrium indeterminacy as well as the dynamic properties around each BGP equilibrium when there are multiple (dual) long-run equilibria (e.g., García-Belenguer, 2007; Greiner, 2003; Park and Philippopoulos, 2004). However, though many of these papers have clearly devoted attention to clarifying the local dynamics around the equilibria, less detail was given to the problem of existence of equilibrium; in particular multiple equilibria (e.g., Alonso-Carrera and Freire-Serén, 2004; Mino, 2004; Pérez and Ruiz, 2007). In fact, multiple equilibria may be less likely to occur than what is generally assumed in the literature.

Hosoya (2012), for instance, fails to realize that positive BGP growth rates may actually correspond to negative levels of consumption along the BGP. By not considering a necessary condition for the existence of long-run equilibria, the author provides two candidates for the steady state, but, since one corresponds to negative levels of consumption throughout time, it turns out to be not relevant.

In this paper, we revisit the growth model with a public health factor proposed by Hosoya (2012) to determine the conditions for the existence of long-run equilibria. Moreover, we try to shed light on the fact that using numeric simulations without considering specific conditions may yield solution candidates that do not correspond to steady-state equilibria.

A correct set of restricted parameters yielding multiple equilibria can however be computed, recovering the author's main findings concerning local dynamics when multiple long-run equilibria exist. For that set, the model accommodates both "growth miracles" and "poverty traps". We show numerical evidence that the emergence of multiple equilibria is unlikely and requires stringent conditions on the parameter values. Over the simulations that yield relevant steady-state equilibria, it can be shown that they always exhibit saddle-path stability. Noteworthy, it is possible that no equilibrium solution exists even in the case where there is only one candidate for equilibrium.

The remainder of the paper is organized as follows. Section 2 briefly replicates the derivations of the original model by Hosoya (2012) up to the two-dimensional system of differential equations that describe the dynamics of the model. Section 3 analyzes the existence of equilibria in the model and their local stability. Section 4 shows numerical simulations to highlight the critical perspective on "growth miracles" in multiple equilibria. Finally, Section 5 is left for some concluding remarks.

2. The model: Brief presentation

To better understand the Hosoya (2012) model, in this section we summarize the respective dynamic optimization problem.

A representative agent chooses the optimal path for consumption $C(t)$:

$$\max_{C} \int_0^\infty \frac{(CH^\sigma)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \sigma \geq 0, \theta > 0 \wedge \theta \neq 1, \rho > 0$$
$$\text{s.t. } \dot{K} = (1-\tau)K^\alpha H^{1-\alpha} - C, \tau, \alpha \in (0,1)$$
$$K(0) = K_0 > 0,$$

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where $H(t)$ is the level of public health infrastructure, $K(t)$ is the level of physical capital, $\dot{K}(t) = \frac{\partial K(t)}{\partial t}$, σ is the weight of public health in the utility function, θ is the inverse of the intertemporal elasticity of substitution, ρ is the discount rate, τ is the income tax rate and α is the share of physical capital in production. The economy produces homogeneous final goods, $Y(t)$. The production function of Y at time t is given by $Y(t) = K(t)^{\alpha}[H(t)L(t)]^{1-\alpha}$, where $L(t)$ is total labor force. The agent maximizes discounted lifetime utility disregarding the effect of public health infrastructure; thus, H is an exogenous variable. The solution to the optimization problem yields the following Euler equation (for simplicity, from now on we omit the time variable unless necessary for clarity):

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left[\alpha(1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} + \sigma(1-\theta)\frac{\dot{H}}{H} - \rho\right]. \tag{1}$$

The dynamics of public health infrastructure are as follows:

$$\dot{H} = \delta\tau K^{\alpha} H^{1-\alpha}, \quad \delta > 0, \tag{2}$$

where δ stands for the technological efficiency parameter of public health creation. Using Eqs. (1) and (2) and the dynamic equation for $\dot{K}$ we get the following system:

$$\frac{\dot{K}}{K} = (1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} - \frac{C}{K},$$

$$\frac{\dot{H}}{H} = \delta\tau\left(\frac{K}{H}\right)^{\alpha}, \tag{3}$$

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left[\delta\sigma\tau(1-\theta)\left(\frac{K}{H}\right)^{\alpha} + \alpha(1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} - \rho\right]. \tag{4}$$

Rewriting the previous tri-dimensional system, assuming $X \equiv C/K$ and $Z \equiv K/H$, we get a two-dimensional one:

$$\frac{\dot{X}}{X} = \frac{1}{\theta}\left[\delta\sigma\tau(1-\theta)Z^{\alpha} + \alpha(1-\tau)Z^{\alpha-1} - \rho\right] - (1-\tau)Z^{\alpha-1} + X,$$

$$\frac{\dot{Z}}{Z} = (1-\tau)Z^{\alpha-1} - X - \delta\tau Z^{\alpha}. \tag{5}$$

Since $C(0)$ is the jump variable, the value of $X(0)$ is not predetermined. Our new control variable is thus X.

3. Equilibria analysis

A steady state implies that $\dot{Z} = 0$ and $\dot{X} = 0$. By construction, this implies that C, K, H and Y grow at the same rate and are all greater than zero, i.e., at the BGP we have $g \equiv \dot{C}/C = \dot{K}/K = \dot{H}/H = \dot{Y}/Y$. Hence, from Eq. (3) we reach:

$$Z_{ss} = \left(\frac{g}{\delta\tau}\right)^{\frac{1}{\alpha}}, \tag{6}$$

which combined with Eq. (4) results in:

$$\underbrace{\alpha(1-\tau)\left(\frac{g}{\delta\tau}\right)^{\frac{\alpha-1}{\alpha}}}_{\equiv \Gamma(g)} = \underbrace{(\theta-\sigma(1-\theta))g + \rho}_{\equiv \Psi(g)} \tag{7}$$

Consider $\Gamma(g)$ and $\Psi(g)$ the LHS and RHS of Eq. (7), respectively. Since Ψ is linear and Γ is strictly convex and decreasing in g, two different scenarios must be tackled:

- Scenario (i): when $\theta < \sigma/(1+\sigma)$, i.e., when Ψ is negatively sloped;
- Scenario (ii): when $\theta \geq \sigma/(1+\sigma)$.

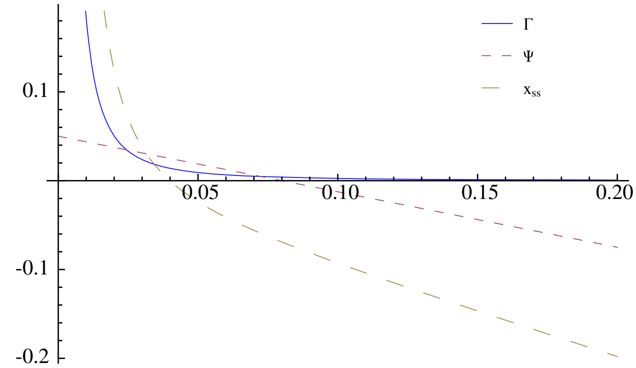

Fig. 1. Two growth equilibria, but the high-growth equilibrium turns out to be non-relevant (since $X_{ss}(g_h) < 0$).

Hosoya (2012) states that in the second scenario there is a unique long-run equilibrium, whereas in the first the emergence of multiple (dual) equilibria is possible.

While this is true, we find that it is also possible that no equilibrium solution exists in both scenarios. Hosoya (2012) seems to have overlooked the fact that finding a positive solution for g in Eq. (7) is not sufficient for an equilibrium solution to exist. In the first scenario, a solution to Eq. (7) may not even exist. This is the case when there is no intersection between Γ and Ψ.

The proposition below provides the necessary condition for the relevance of the equilibria.

Proposition 1. *Solving for g in* Eq. (7) *only yields a relevant BGP if:*

$$X_{ss} \equiv (1-\tau)\left(\frac{g}{\delta\tau}\right)^{\frac{\alpha-1}{\alpha}} - g \geq 0. \tag{8}$$

Proof. Consider a strictly positive solution for g from Eq. (7). Then it follows a positive steady state level for Z from Eq. (6), which means that both K and H are positive at steady state. Substituting the expression for Z_{ss} in Eq. (5) and setting $\dot{Z} = 0$, we recover Eq. (8).

If X is negative, then also consumption C is negative since K is always positive; $C < 0$ has no economic sense. Hence, g can only be a growth rate corresponding to a relevant equilibrium for X and Z if $X \geq 0$. □

The function X_{ss} in Eq. (8) tends to infinity as g goes to zero and is negative for a high enough g.[1] Furthermore, it is continuous and strictly decreasing in g. Thus, we can conclude that X_{ss} is negative for a $g > \overline{g}$ such that $X_{ss}(\overline{g}) = 0$.

Moreover, since it is strictly decreasing, the higher the economic growth rate, the less likely it is that it will correspond to a relevant equilibrium. In that sense, in the presence of possible multiple equilibria, if one of them turns out to be non-relevant, it will be the one that corresponds to the high-growth equilibrium.

This result has strong implications in recent economic growth literature on multiple equilibria. The main conclusion is that the so called "growth miracles" (high-growth equilibrium, g_h) are actually less likely to occur compared to "poverty traps" (low-growth equilibrium, g_l), in which economic growth rates at the BGP are lower. This stems from the imposition of an additional necessary condition on the existence of equilibria through the non-negativeness of consumption at the BGP.

Remark 1. A numerical approach for computing equilibria by directly setting $\dot{X} = 0$ and $\dot{Z} = 0$ in Eq. (5) would seem to render our analysis somewhat trivial as the condition for non-negativeness would naturally be imposed on the resulting values of X. However, this method makes it difficult to determine the number of equilibria. As a result, solving for equilibria in this model by finding the BGP growth rates first is a more

[1] Indeed, we have $\lim_{g \to \infty} X_{ss}(g) = -\infty$.