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Multifractal detrended cross-correlation analysis between the Chinese stock market and surrounding stock markets

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ABSTRACT

In this paper, we investigate the cross-correlations between the stock market in China and markets in Japan, South Korea and Hong Kong. We use not only the qualitative analysis of the cross-correlation test, but also the quantitative analysis of the MF-X-DFA. Our findings confirm the existence of cross-correlations between the stock market in China and markets in Japan, South Korea and Hong Kong, which have strongly multifractal features. We find that the cross-correlations display the characteristic of multifractality in the short term. Moreover, the cross-correlations of small fluctuations are persistent and those of large fluctuations are anti-persistent in the short term, while the cross-correlations of all kinds of fluctuations are persistent in the long term. Furthermore, based on the multifractal spectrum, we also find that the multifractality of cross-correlation between stock markets in China and Japan are stronger than those between China and South Korea, as well as between China and Hong Kong.

1. Introduction

Since Hurst [1,2] derived the long memory of the hydrological time series from tide data, and Mandelbrot et al. [3] laid the scrutinized mathematical foundation by introducing the Brownian movement and the Sectioning conception, research of long memory has aroused widespread interests. In the last two decades, the methods of examining long memory are widely applied to financial markets. For example, Peters [4] adopted the Rescaled Range analysis (R/S) in examining long memory in historical stock markets. Peters [4] proved that the returns almost go with the property of high peak and fat tail and found that financial markets in the US, the UK and the Japan have embodied the long-range auto-correlation which implied the market inefficiency. Using the R/S method, Cajueiro and Tabak [5] studied the long-range auto-correlations in emerging markets and found that the markets became more and more efficient over time. Barkoulas et al. [6] applied the R/S in research of the interest rates and the stock market and obtain a similar conclusion. Lo [7] contended that R/S seemed to be rather sensitive to the short term auto-correlation and the non-stationarity, which is likely to lead to a biased estimation of long memory parameters. In order to overcome the drawbacks, Peng et al. [8] put forward the DFA in studying the fractal structure of molecular chains in deoxyribonucleic acid (DNA). Since then, DFA and its multifractal generalization, MF-DFA [9] have been widely used to detect the long-range auto-correlations in financial markets, including the stock markets [10,11], foreign exchange market [12,13], and gold market [14].

It has been found that financial time series are always cross-correlated [15–22]. To quantify power-law cross-correlations in non-stationary time series, Podobnil and Stanley [23] extend the DFA into the DXA method. Subsequently, Zhou [24] proposed multifractal detrended cross-correlation analysis (MF-X-DFA) by combining MF-DFA and DXA approaches. Since

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then, DXA and MF-DXA method have been widely used in diverse fields including financial data [25,26], traffic flows [27–29], sunspot numbers and river flow fluctuations [30], and meteorological data [31]. Being of our interest, this method is also applied to Chinese stock and commodity markets. For example, Wang et al. [32] studied the cross-correlations between the A and B shares of the Chinese stock market. Cao et al. [33] discussed the cross-correlations between the Chinese stock market and the foreign exchange market of Chinese yuan. Li et al. [34] researched the cross-correlation properties of agricultural futures markets between the China and US, and found that the cross-correlations are significantly multifractal. Yuan et al. [35] found that the cross-correlation between the Chinese stock price and trading volume is multifractal. As the deep development of the methodology, Jiang and Zhou [36] introduced MF-X-DMA which is based on MF-DMA [37] and DMA [38]. Kristoufek proposed MF-HXA [39] based on the height–height correlation analysis of Barabasi and Vicsek [40] and Hedayatifar et al. [41].

This paper is focused upon the cross-correlations between the stock market in China and markets in Japan, South Korea and Hong Kong by means of MF-X-DFA. Our contributions are fourfold. First, this paper is the first study which studies multifractal cross-correlations between the Chinese stock market and surrounding stock markets. Second, we not only qualitatively analyze the cross-correlations employing the statistics proposed by Podobnik et al. [42] and Podobnik et al. [43], but also quantitatively study the cross-correlations using the MF-X-DFA method. Third, we use the rolling windows to investigate the time-varying features of multifractal cross-correlations. Finally, we study the market efficiency of the Chinese stock market using the MF-X-DFA method and the technique of rolling windows.

This paper is organized as follows. Section 2 mainly focuses upon the description of MF-X-DFA. Section 3 describes the data of four stock markets of interests. Section 4 provides the empirical analysis. Section 5 discusses the results based on rolling windows and Section 6 concludes.

2. Methodology

Assume that there are two series $x(i)$ and $y(i)(i = 1, 2, \dots, N)$, let us introduce the MF-X-DFA method as follows.

Step 1. Construct the profile

$$X(i) = \sum_{t=1}^{i}(x(t) - \bar{x}), \qquad Y(i) = \sum_{t=1}^{i}(y(t) - \bar{y}) \tag{1}$$

where, $\bar{x}$ and $\bar{y}$ denote the average of the two whole time series $x(i)$ and $y(i)$.

Step 2. The profiles $X(i)$ and $Y(i)$ are divided into $N_s = [N/s]$ non-overlapping windows (or segments) of equal length s. Since the length N is not always a multiple of the considered time scale s. In order to not discard the Section of series, the same procedure is repeated starting from the opposite end of each profile. Thus, $2N_s$ non-overlapping windows are obtained together.

Step 3. The local trends $X^{\nu}(i)$ and $Y^{\nu}(i)$ for each segment $\nu(\nu = 1, 2, 3, \dots, 2N_s)$ are evaluated by least squares fits of the data, then the detrended covariance is determined by

$$F^2(s, \upsilon) = \frac{1}{s}\sum_{i=1}^{t} |X((\upsilon - 1)s + i) - X^{\upsilon}(i)| \bullet |Y((\upsilon - 1)s + i) - Y^{\upsilon}(i)| \tag{2}$$

for each segment ν, $\nu = 1, 2, \dots, N_s$ and

$$F^2(s, \upsilon) = \frac{1}{s}\sum_{i=1}^{t} |X(N - (\upsilon - N_s)s + i) - X^{\upsilon}(i)| \bullet |Y(N - (\upsilon - N_s)s + i) - Y^{\upsilon}(i)| \tag{3}$$

for each segment ν, $\nu = N_s + 1, N_s + 1, \dots, 2N_s$. Then the trends $X\nu(i)$ and $Y\nu(i)$ denote the fitting polynomial with order m in each segment ν.

Step 4. qth-order the fluctuation function as follows

$$F_q(s) = \left[\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}[F^2(s, \nu)]^{q/2}\right]^{1/q}. \tag{4}$$

If $q \neq 0$, then

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{\nu=1}^{2N_s}\ln[F^2(s, \nu)]\right\}. \tag{5}$$

When $q = 2$, MF-X-DFA is the standard of the DXA.