Economic Modelling 33 (2013) 552–559

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Stochastic dominance relationships between stock and stock index futures markets: International evidence

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ARTICLE INFO

Article history:
Accepted 30 April 2013

JEL classification:
C12
G14
G15

Keywords:
Stochastic dominance
Risk averter
Risk seeker
Stock
Index futures

ABSTRACT

In this paper, we first modify the stochastic dominance (SD) test for risk averters proposed by Davidson and Duclos (2000) to be the SD test for risk seekers. We then adopt both tests to examine the SD relationships between stock indices and their corresponding index futures for 10 countries. The sample contains data from 6 developed countries and 4 developing countries. The study proposes that there should be no SD relationship between spot and futures markets in developed financial markets in which arbitrage opportunities (both pure and quasi) are rare and short-lived. However, we expect that SD relationships could be found in emerging financial markets that have more impediments to arbitrage. Consistent with this conjecture, our study finds that there are no SD relationships between spot and futures markets in the mature market sample, implying that these markets could be efficient. However, for the emerging markets, spot dominates futures for risk averters, while futures dominate spot for risk seekers in the second- and third-order SD. These results indicate that there are potential gains in expected utilities for risk averters (seekers) if they switch their investment from futures (spot) to spot (futures) in the emerging markets.

1. Introduction

In an efficient market, the force of arbitrage ensures that a futures contract and the price of its underlying cash asset move in lock-step and a movement in either asset, say, the futures, will trigger spontaneous movement in the corresponding spot market. As a result, there should be no arbitrage opportunity between futures and the underlying cash asset and they should be perfect substitutes for investors. However, real market factors, such as execution risk, market liquidity, participation of astute and highly capitalized arbitrageurs, transaction costs, and institutional constraints like restrictions on short-selling cash assets, impede arbitrage and de-couple the two assets.

Arbitrage inefficiency between futures and spot has been revealed in different forms and by many studies. Previous studies have examined the efficiency issue by testing the existence of pure arbitrage opportunities and a lead–lag relationship between the two markets.[1] However, these two approaches are highly related because imperfect correlation between the two markets is necessary for the existence of pure arbitrage opportunities. The tests on pure arbitrage opportunities are considered to be "weak" form tests.

This study proposes a "strong" form test of market efficiency via stochastic dominance (SD) under both risk aversion and risk seeking assumptions. The complete set of SD tests allows the study to detect the existence of "quasi" arbitrage possibilities even in the absence of pure arbitrage opportunities. Therefore, the SD tests are better in the sense that they can pick up a less apparent form of inefficiency between the two markets.

Due to reasons such as a lower degree of institutional participation, less efficient trading systems that prevent efficient trade execution, less informed investors, and less arbitrage capital in emerging financial markets, we would expect that it is more likely to discover a dominance relationship between the two assets in emerging markets than in mature markets. The study tests the above proposition by examining a group of 6 developed markets and another group of 4 emerging markets. Consistent with our conjecture, the study finds that there are significant SD relationships in the 4 emerging markets, indicating the existence of at least quasi arbitrage opportunities in these countries. On the other hand, no SD relationship is found in the developed market sample.

The rest of this paper is organized as follows. We will briefly discuss the theory of SD for risk averters and risk seekers in Section 2. Section 3 introduces the Davidson and Duclos test (hereafter DD

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[1] Many studies have investigated the lead–lag relationship between the two markets by applying various parametric econometric models; see, for example, Kawaller et al. (1987), Stoll and Whaley (1990), Chan (1992), Zhong et al. (2004), and many others. However, their analyses are usually conducted under certain assumptions about market return distributions, which are well-known to be violated in reality.

0264-9993/$ – see front matter
http://dx.doi.org/10.1016/j.econmod.2013.04.049

test, 2000) for SD and discusses test implementation issues. Section 4 describes the data set and presents descriptive statistics. The SD empirical results are presented and analyzed in Section 5. Section 6 contains some concluding remarks.

2. SD theory

SD theory, initially developed by Hadar and Russell (1969), Hanoch and Levy (1969), and many others is one of the most useful tools for ranking investment prospects in investment decision-making under uncertainty. Let F and G be the cumulative distribution functions (CDFs), and f and g be the corresponding probability density functions (PDFs) of two investments, X and Y, respectively, with common support $[a,b]$ where $a < b$. Define

$$H_0^A = H_0^D = h,\ \ H_j^A(x) = \int_a^x H_{j-1}^A(t)\,dt \ \text{ and }\ H_j^D(x) = \int_x^b H_{j-1}^D(t)\,dt \qquad (1)$$

for $h = f$, and g; $H = F$, and G; and $j = 1$, 2, and 3. We call the integral H_j^A the j^{th} order ascending cumulative distribution function (ACDF) and the integral H_j^D the j^{th} order descending cumulative distribution function (DCDF) for $H = F$ and G and for $j = 1$, 2 and 3.

We first state definition the first-, second- and third-order ascending SD (ASD)[2] for risk averters, denoted as FASD, SASD, and TASD, respectively in the following (see, for example, Fishburn, 1964; Quirk and Saposnik, 1962):

Definition 1. *X dominates Y by FASD (SASD, TASD), denoted by $X \succ_1 Y$ ($X \succ_2 Y$, $X \succ_3 Y$) if and only if $F_1^A(x) \le G_1^A(x)$($F_2^A(x) \le G_2^A(x)$, $F_3^A(x) \le G_3^A(x)$), for all possible returns x, and the strict inequality holds for at least one value of x.*

Wong and Li (1999), Li and Wong (1999), and others show that SD rules apply to risk seekers, with the preferences reversed to those of risk averters under certain conditions.[3] Whereas SD for risk averters works with the ACDF, which counts from the worst return to the best return, SD for risk seekers works with the DCDF, which counts from the best return descending to the worst return (Post and Levy, 2005; Stoyan, 1983; Wong, 2007). Hence, SD for risk seekers could be called descending SD (DSD). We have the following definition for DSD (see, for example, Meyer, 1977; Stoyan, 1983).

Definition 2. *X dominates Y by FDSD (SDSD, TDSD) denoted by $X \succ^1 Y$($X \succ^2 Y$, $X \succ^3 Y$) if and only if $F_1^D(x) \ge G_1^D(x)$ ($F_2^D(x) \ge G_2^D(x)$, $F_3^D(x) \ge G_3^D(x)$), for all possible returns x, the strict inequality holds for at least one value of x; where FDSD (SDSD, TDSD) stands for first-order (second-order, third-order) descending SD.*

We note that in Definitions 1 and 2, the condition "$\mu_x \ge \mu_y$" needs to be included for the third-order ASD and DSD (R.H. Chan et al., 2012). We define utility functions for risk averters, U_j^A, and risk seekers, U_j^D, as shown in the following:

Defination 3. For $j = 1$, 2, and 3, $u \in U_j^A$ or U_j^D is a utility function such that

$$U_j^A = \left\{u : (-1)^i u^i \le 0, i = 1, \ldots j\right\} \ \text{ and }\ U_j^D = \left\{u : (-1)^i u^i \ge 0, i = 1, \ldots j\right\}.$$

Investors with utility u in U_1^A or U_1^D are nonsatiated (prefer more to less); those with utility u in U_2^A are nonsatiated and risk-averse; and those with utility u in U_3^A are nonsatiated and risk-averse and have decreasing absolute risk aversion (DARA). On the other hand, investors with utility u in U_2^D are nonsatiated and risk-seeking, while those with utility u in U_3^D are non-satiated and risk-seeking and have increasing absolute risk-seeking. Thus, we call investors with utility u in U_j^A the jth order of risk averters and investors with utility u in U_j^D the jth order of risk seekers. SD analysis is important because investigating the SD relationship across different financial assets is equivalent to examining the choice of assets by utility maximization under SD theory. To be precise, if $X \succ_j Y$, then $E[u(X)] - E[u(Y)] > 0$ for any risk averter with utility u in U_j^A and if $X \succ^j Y$, then $E[u(X)] - E[u(Y)] > 0$ for any risk seeker with utility u in U_j^D for any $j = 1$, 2, and 3. See Li and Wong (1999) for more information. The SD theory could be extended to a range of non-expected utility. See Wong and Ma (2008) and the references therein for more information.

We also note that a hierarchical relationship exists (C.Y. Chan et al., 2012; Levy, 1992) in ASD (DSD): FASD (FDSD) implies SASD (SDSD), which in turn implies TASD (TDSD). However, the converse is not true: the existence of SASD (SDSD) does not imply the existence of FASD (FDSD). Likewise, the existence of TASD (TDSD) does not imply the existence of SASD (SDSD) or FASD (FDSD).

An individual chooses between F and G in accordance with a consistent set of preferences satisfying the von Neumann and Morgenstern (1944) consistency properties. Accordingly, F is (strictly) preferred to G, or equivalently, X is (strictly) preferred to Y if

$$\Delta E_u \equiv E[u(X)] - E[u(Y)] \ge 0 (> 0)$$

where $E[u(X)] \equiv \int_a^b u(x)dF(x)$ and $E[u(Y)] \equiv \int_a^b u(x)dG(x)$.

To make a choice between two assets X and Y, risk averters will compare their corresponding jth order ASD integrals and choose X if F_j^A is smaller for $j = 1$, 2, and 3. On the other hand, risk seekers will compare their corresponding jth DSD integrals and choose X if F_j^D is bigger (Wong and Chan, 2008).

Finally, we note that investment X stochastically dominates investment Y by FASD or FDSD, if and only if there is a pure arbitrage opportunity between X and Y, such that investors with increasing utility will increase their expected wealth as well as their expected utilities if their investments are shifted from Y to X. They could make big profits by setting up zero dollar portfolios to exploit this opportunity. On the other hand, if investment X stochastically dominates investment Y by SASD or TASD (SDSD or TDSD), risk averters (seekers) will increase their expected utilities but not their expected wealth if their investments are shifted from Y to X (Wong et al., 2008).

3. Davidson and Duclos (DD) test

The early work of Beach and Davidson (1983) examines dominance at the first order. More recently, several methods have been proposed for testing for SD of other orders (Bai et al., 2011; Barrett and Donald, 2003; Davidson and Duclos, 2000; Linton et al., 2005). Some studies like Wei and Zhang (2003), Tse and Zhang (2004), and Lean et al. (2008) show that the DD test is powerful and parsimonious. Hence, we adopt DD statistics and the DD test for the empirical work in this paper.

3.1. Davidson and Duclos (DD) test for risk averters

Let $\{(f_i, s_i)\}$ ($i = 1, \ldots, n$) be n pairs of observations for futures and spot, respectively, drawn from the random variables X and Y, with their integrals F_j^A and G_j^A, respectively, defined in (1) for $j = 1$, 2,

[2] We call it ascending SD, since its integrals count from the worst return ascending to the best return.

[3] Readers may refer to Lemma 3 and Theorems 7, 10, and 12 in Li and Wong (1999) and Theorem 1 and Corollary 1 in Wong and Li (1999) for more information on the relationship of preferences between risk averters and risk seekers.