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# Multifractal cross-correlation spectra analysis on Chinese stock markets

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## HIGHLIGHTS

- Legendre spectrum and large deviations spectrum are extended.
- The multifractal structure of stock return and volatility series is presented.
- Local detrended covariance with broad distribution leads to multifractality.

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## ABSTRACT

In this paper, the long-range cross-correlation of Chinese stock indices is systematically studied. The multifractal detrended cross-correlation analysis (MF-DXA) appears to be one of the most effective methods in detecting long-range cross-correlation of two non-stationary variables. The Legendre spectrum and the large deviations spectrum are extended to the cross-correlation case, so as to present multifractal structure of stock return and volatility series. It is characterized of the multifractality in Chinese stock markets, partly due to clusters of local detrended covariance with large and small magnitudes.



## 1. Introduction

The studies on complex systems have been the focus of many researchers in recent decades [1–3]. Complex systems are systems that exhibit several defining properties, including: (i) feedback loops, where change in a variable results in either a positive feedback or a negative feedback of that change; (ii) many interdependent variables; (iii) chaotic behavior; (iv) a non-Gaussian distribution of outputs. In most cases, complex systems are characterized with such one or more properties as nonlinearity, non-stationarity, noises, trivial oscillations, and thus become difficult for researchers to analyze. Furthermore, the self-similarity and the rule of scale invariance have been found in many real-world systems, which indicates the presence of fractal or multifractal structure [4–9].

As an example of real-world systems, the financial markets exhibit very complex dynamics and in recent years have been the focus of some physicists' attempts to apply statistical mechanics to economic problems. The financial markets are open systems in which many subunits interact nonlinearly in the presence of feedback and many records of the non-stationary time series have been investigated. Multifractality is one of the well-known stylized facts which characterizes non-trivial properties of financial time series. Many researchers have suggested that multifractality is a pervasive characteristic of stock prices [10–14]. In quantitative finance, volatility is one of the most important risk measures since it corresponds to the conditional variance associated with price fluctuations. A well-known fact is that volatility fluctuations are organized into

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persistent clusters [15]. It is previously shown that the multifractality degree is related to the stage of market development. More precisely, higher multifractality is associated with a less developed market [16,17].

It is a rather common behavior of real-world systems that they do not operate independently. Cross-correlation of variables between relevant systems is thus frequently observed. Examples include the heart rate and respiratory rate in biological identities, the air temperature and air humidity in atmospheric environment, the road occupancy and vehicle speed in traffic network, the trading price and trading volume in stock markets, etc. [18–26]. In this paper, we focus to study the existence of cross-correlation on Chinese stock prices, mainly on the stock return and volatility series. We refer to three methods, to estimate the long-range cross-correlation of two variables. They include the linear cross-correlation function, the multifractal detrended cross-correlation analysis (MF-DXA), and the multifractal height cross-correlation analysis (MF-HXA). These methods are applied to the artificial variables, the theoretical results of which can be prior inferred, to determine which one is more effective. And for two non-stationary variables, the MF-DXA method seems to be the most effective one, both for monofractal and multifractal cases.

Another fundamental concern is to study the possible fractal structure of the long-range cross-correlation. The Legendre spectrum and the large deviations spectrum were extensively studied on the auto-correlation analysis of single variable (Fig. 4). Here we extend them to measure the structure of cross-correlation. Although different measurements are estimated from different views, they present rather similar results. It is a rather important practice for statistical physics methods applied in real-world systems, as only if at least two independent methods consistently reveal the structure of cross-correlation, one can be sure that the variables are indeed monofractal or multifractal. In Chinese stock markets, the multifractality of return and volatility series is characterized.

## 2. Long-range cross-correlation analysis

### 2.1. Linear cross-correlation function

Consider two stationary variables $x$ and $y$, with equal length $N$. Their linear cross-correlation function is described as follows,

$$C_s(x, y) = \frac{1}{N - s} \sum_{i=1}^{N-s} \{[x_i - \langle x \rangle][y_{i+s} - \langle y \rangle]\}, \tag{1}$$

where $\langle \cdots \rangle$ denotes the mean value. $s$ represents the time delay between two variables, i.e., to consider the relationship between variable $x$ at time $i$ and variable $y$ at time $i + s$.

If two variables are linearly independent, $C_s(x, y) \equiv 0$ for any $s \in \{0, 1, 2, \ldots\}$. When two variables are short-range cross-correlated, there exists a positive integer $m$ such that $C_s(x, y) \neq 0$ for $s \in \{0, 1, \ldots, m\}$ while $C_s(x, y) \equiv 0$ for $s > m$. In real-world systems, $C_s(x, y)$ is almost impossible to attain 0 while can be treated as 0 if it falls in the confidence interval at a pre-defined significant level, as indicated in Fig. 1. Another case is $\sum_s C_s(x, y) \to \infty$ if $N \to \infty$, then two variables are considered as long-range cross-correlated. If the long-range cross-correlation exists between two variables,

$$C_s(x, y) \sim s^{-\gamma}, \tag{2}$$

where $\gamma$ is the cross-correlation scaling exponent, and particularly, falls in the range $0 < \gamma < 1$. $\gamma$ describes the rate of decay. A larger $\gamma$ represents a weaker strength of cross-correlation along $s$, and vice versa.

However, it is rather limited of linear cross-correlation function to describe the cross-correlation, as (i) it only considers the linear cross-correlation while neglects the nonlinear part; (ii) it is limited to cope with stationary series, since $\langle x \rangle$ or $\langle y \rangle$ may change with time for non-stationary series, (iii) it usually oscillates abruptly at large $s$ that makes it difficult to accurately estimate the value of $\gamma$, and (iv) it fails to characterize the structure of cross-correlation at different statistical moments.

There exists other methods to measure the long-range cross-correlation of stationary variables, e.g., the power spectrum, which is the Fourier transform of the covariance function [28]. While, it also oscillates abruptly at large frequencies, thus seems uncertainty to present the scaling property.

### 2.2. Multifractal detrended cross-correlation analysis (MF-DXA)

Consider two variables $x$ and $y$ that are non-stationary, i.e. their statistical moments (e.g. the mean value, variance) may change with time. Using non-stationary variable often produces unreliable and spurious results and leads to poor understanding and forecasting. The solution to the problem is to transform the variable so that it becomes stationary, typically by the differencing and detrending procedures. If the non-stationary process is a random walk with or without a drift, it is transformed to stationary process by differencing. On the other hand, if the variable analyzed exhibits a deterministic trend, the spurious results can be avoided by detrending. Sometimes the non-stationary variable may combine a stochastic and deterministic trend at the same time and to avoid obtaining misleading results both differencing and detrending should be