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Humps in the volatility structure of the crude oil futures market: New evidence

Carl Chiarella [a], Boda Kang [a], Christina Sklibosios Nikitopoulos [a,*], Thuy-Duong Tô [b]

[a] University of Technology Sydney, Finance Discipline Group, UTS Business School, PO Box 123, Broadway, NSW 2007, Australia
[b] University of New South Wales, Australian School of Business, Sydney NSW 2052, Australia

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This paper analyses the volatility structure of commodity derivatives markets. The model encompasses hump-shaped, unspanned stochastic volatility, which entails a finite-dimensional affine model for the commodity futures curve and quasi-analytical prices for options on commodity futures. Using an extensive database of crude oil futures and futures options spanning 21 years, we find the presence of hump-shaped, partially spanned stochastic volatility in the crude oil market. The hump shaped feature is more pronounced when the market is more volatile, and delivers better pricing as well as hedging performance under various dynamic factor hedging schemes.

1. Introduction

Commodity derivatives serve the very important role of helping to manage the volatility of commodity prices. Apart from hedgers, the volatility of commodity prices is also of keen interest to speculators, who have become more dominant in these markets in recent years, see Barone-Adesi et al. (2010) and Cifarelli and Paladino (2010). However, these derivatives have their own volatility, of which the understanding and management is of paramount importance. In this paper, we will provide a tractable model for this volatility, and carry out empirical analysis for the most liquid commodity derivative market, namely the crude oil market.

The model used in this paper focuses directly on the volatility of derivatives. It is set up under the Heath et al. (1992) framework that treats the entire term structure of futures prices as the primary modelling element. Due to the standard feature that commodity futures prices are martingales under the risk-neutral measure, the model is completely identified by the volatility of futures prices and the initial forward curve. We model this volatility as a multifactor stochastic volatility, which may be partially unspanned by the futures contracts. Spot commodity prices are uniquely determined without the need to specify the dynamics of the convenience yield. Option prices can be obtained quasi-analytically[1] and complex derivative prices can be determined via simulation.

Commodity derivatives have been previously studied under the Heath et al. (1992) framework. However, previous works such as those of Miltersen and Schwartz (1998), Clewlow and Strickland (2000) and Miltersen (2003) as well as the convenience yield models of Gibson and Schwartz (1990) and Korn (2005) were restricted to deterministic volatility. Schwartz and Trolle (2009a) extended the literature significantly by providing empirical evidence that stochastic volatility models are superior to the deterministic volatility models as the latter are not capable of accommodating the very important feature of unspanned volatility in the commodities markets. In addition, Clewlow and Strickland (1999) have demonstrated that convenience yield models can be converted to forward models which are a class of models employed in our study. However, there are two differences between this paper and the Schwartz and Trolle (2009a) paper. First, Schwartz and Trolle (2009a) start by modelling the spot commodity and convenience yield. Convenience yield is unobservable and therefore its modelling adds complexity to model assumptions and estimation. Moreover, sensitivity analysis has to rely on applying shocks to this unobserved convenience yield, which makes it less intuitive. Second, the volatility function in the Schwartz and Trolle (2009a) paper has an exponential decaying

* Corresponding author. Tel.: +61 2 9514 7768.
E-mail addresses: carl.chiarella@uts.edu.au (C. Chiarella), boda.kang@uts.edu.au (B. Kang), christina.nikitopoulos@uts.edu.au (C.S. Nikitopoulos), td.to@unsw.edu.au (T.-D. Tô).

[1] Empirical studies that measure and forecast volatility in commodities markets, see for instance Agnolucci (2009), suggest that more advanced option pricing models could provide better results.

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form, predicting that long term contracts will always be less volatile than short term contracts. Our model, on the other hand, uses a hump-shaped volatility (which can be reduced to an exponential decaying one), and therefore allows for increasing volatility at the short end of the curve.

The model in this paper falls under the generic framework provided by Andersen (2010) for the construction of Markovian models for commodity derivatives. As an extension to his work, we provide full results for models that allow for hump-shaped, unspanned stochastic volatility. A hump is an important factor in other markets, such as interest rate markets, see for example Litterman et al. (1991), Dai and Singleton (2000) and Bekaert et al. (2001). However, limited evidence exists in the crude oil market. As far as we are aware, this feature has only been studied in the working paper version of Schwartz and Trolle (2009a). It is reported that a hump shaped volatility function had been tried, but resulted in very similar estimates and almost indistinguishable price performance compared to the exponential volatility function. We will re-examine the volatility structure of the crude oil derivatives market. We use a larger panel dataset of crude oil futures and options traded on the NYMEX, spanning 21 years from 1 January 1990 to 31 December 2010. We find that a three-factor stochastic volatility model works well. Two of the volatility functions have a hump shape that cannot be captured by the exponential decaying specification. We also find that the hump shaped volatility matters a lot more when the market is volatile than when the market is relatively stable. The extent to which the volatility can be spanned by futures contracts varies over time, with the lowest spanning being in the recent period of 2006–2010.

The fact that volatility in the market cannot be spanned by futures contracts highlights the importance of options for hedging purposes. We analyse the hedging of straddle contracts, the pricing of which is highly sensitive to volatility. Given the multifactor nature of the model, factor hedging is employed. Factor hedging has been used successfully for deterministic and local volatility models,[2] such as in Clewlow and Strickland (2000) or Fan et al. (2003). We expand the method to hedge the random shocks coming from stochastic volatility. We show that the hedging performance increases dramatically when options contracts are added to the hedging instrument set. The hedging performance is measured under various different factor hedging schemes, from delta-neutral to delta-vega and delta-gamma neutral.

An alternative approach to the HJM framework is modelling the spot commodity prices directly. A representative list of relevant literature would include Gibson and Schwartz (1990), Litzenberger and Rabinowitz (1995), Schwartz (1997), Hilliard and Reis (1998), Casassus and Collin-Dufresne (2005) and Fusai et al. (2008). These models have been successful in depicting essential and critical features of distinct commodity market prices, for instance, the mean-reversion of the agricultural commodity market, the seasonality of the natural gas market, the spikes and regime switching of the electricity market and the inverse leverage in the oil market. The disadvantage of the spot commodity models is the requirement to specify and estimate the unobservable convenience yield. The futures prices are then determined endogenously. In addition, some spot commodity models cannot accommodate unspanned stochastic volatility,[3] a feature that can be naturally embedded in HJM models.

The paper is organized as follows. Section 2 presents a generalised unspanned stochastic volatility model for pricing commodity derivatives within the HJM framework. Section 3 describes and analyses the data for crude oil derivatives and explains the estimation algorithm. Section 4 presents the results. Section 5 examines the hedging performance. Section 6 concludes. Technical details are presented in the Appendix.

[2] Local volatility models refer to models where there is a dependence between volatility and the level of the state variables.

[3] See the discussion in Collin-Dufresne and Goldstein (2002) for example.

2. The HJM framework for commodity futures prices

We consider a filtered probability space $(\Omega, \mathcal{A}_T, \underline{\mathcal{A}}, P)$, $T \in (0,\infty)$ with $\underline{\mathcal{A}} = (\mathcal{A}_t)_{t\in[0,T]}$, satisfying the usual conditions.[4] We introduce $\mathbf{V} = \{\mathbf{V_t},\ t \in [0,T],\}$ a generic stochastic volatility process modelling the uncertainty in the commodity market. We denote as $F(t,T,\mathbf{V_t})$, the futures price of the commodity at time $t \geq 0$, for delivery at time T, (for all maturities $T \geq t$). Consequently, the spot price at time t of the underlying commodity, denoted as $S(t,\mathbf{V_t})$ satisfies $S(t,\mathbf{V_t}) = F(t,t,\mathbf{V_t})$, $t \in [0,T]$. The futures price process is equal to the expected future commodity spot price under an equivalent risk-neutral probability measure Q, see Duffie (2001), namely

$$F(t,T,\mathbf{V_t}) = \mathbb{E}^Q[S(T,\mathbf{V_T})|A_t].$$

This leads to the well-known result that the futures price of a commodity is a martingale under the risk-neutral measure, thus the commodity futures price process follows a driftless stochastic differential equation. Let $W(t) = \{W_1(t),\ \ldots,\ W_n(t)\}$ be an n-dimensional Wiener process driving the commodity futures prices and $W^V(t) = \{W_1^V(t),\ \ldots,\ W_n^V(t)\}$ be the n-dimensional Wiener process driving the stochastic volatility process $\mathbf{V}_t$, for all $t \in [0,T]$.[5]

Assumption 1. The commodity futures price process follows a driftless stochastic differential equation under the risk-neutral measure of the form

$$\frac{dF(t,T,\mathbf{V_t})}{F(t,T,\mathbf{V_t})} = \sum_{i=1}^{n} \sigma_i(t,T,\mathbf{V_t}) dW_i(t), \tag{1}$$

where $\sigma_i(t,T,\mathbf{V_t})$ are the $\mathcal{A}$-adapted futures price volatility processes, for all $T > t$. The volatility process $\mathbf{V_t} = \{\mathbf{V_t^1},\ \ldots \mathbf{V_t^n}\}$ is an n-dimensional well-behaved Markovian process evolving as

$$d\mathbf{V_t}^i = a_i^V(t,\mathbf{V_t})dt + \sigma_i^V(t,\mathbf{V_t})dW_i^V(t), \tag{2}$$

for $i = 1,\ \ldots,\ n$, where $a_i^V(t,\mathbf{V_t})$, $\sigma_i^V(t,\mathbf{V_t})$ are $\mathcal{A}$-adapted stochastic processes and

$$\mathbb{E}^Q\left[dW_i(t) \cdot dW_j^V(t)\right] = \begin{cases} \rho_i dt, & i = j; \\ 0, & i \neq j. \end{cases} \tag{3}$$

Assume that all the above processes are $\mathcal{A}$-adapted bounded processes with drifts and diffusions that are regular and predictable so that the proposed SDEs admit unique strong solutions. The proposed volatility specification expresses naturally the feature of unspanned stochastic volatility in the model. The correlation structure of the innovations determines the extent to which the stochastic volatility is unspanned. If the Wiener processes $W_i(t)$ are uncorrelated with $W_i^V(t)$ then the volatility risk is unhedgeable by futures contracts. When the Wiener processes $W_i(t)$ are correlated with $W_i^V(t)$, then the volatility risk can be partially spanned by the futures contracts. Thus the volatility risk (and consequently options on futures contracts) cannot be completely hedged by using only futures contracts.

Conveniently, the system of Eqs. (1) and (2) can be expressed in terms of independent Wiener processes. By considering the

[4] The usual conditions satisfied by a filtered complete probability space are: (a) $\mathcal{F}_0$ contains all the $\mathbb{P}$-null sets of $\mathcal{F}$ and (b) the filtration is right continuous. See Protter (2004) for technical details.

[5] We essentially assume that the filtration $\mathcal{A}_t$ includes $\mathcal{A}_t = \mathcal{A}_t^f \vee \mathcal{A}_t^V$, where

$$\left(\mathcal{A}_t^f\right)_{t\geq 0} = \{\sigma(W(s) : 0 \leq s \leq t)\}_{t\geq 0},$$
$$\left(\mathcal{A}_t^V\right)_{t\geq 0} = \left\{\sigma\left(W^V(s) : 0 \leq s \leq t\right)\right\}_{t\geq 0}.$$