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Trading strategies with partial access to the derivatives market

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ABSTRACT

This research analyzes trading strategies with derivatives when there are several assets and risk factors. We investigate portfolio improvement if investors have full and partial access to the derivatives markets, i.e. situations in which derivatives are written on some but not all stocks or risk factors traded on the market. The focus is on markets with jump risk. In these markets the choice of optimal exposures to jump and diffusion risk is linked. In a numerical application we study the potential benefit from adding derivatives to the market. It turns out that e.g. diffusion correlation and volatility or jump sizes may have a significant impact on the benefit of a new derivative product even if market prices of risk remain unchanged. Given the structure of risk investors may have different preferences for making risk factors tradable. Utility gains provided by new derivatives may be both increasing or decreasing depending on the type of contract added.

1. Introduction

Optimal portfolio choice certainly belongs to one of the most extensively studied problems in finance. Merton (1969, 1971, 1973) considers continuous time economies in which individuals dynamically adjust portfolio positions in order to maximize expected utility. More recent contributions addressing allocation include e.g. Liu et al. (2003), Wu (2003), Das and Uppal (2004), Munk and Sorensen (2004), Rudolf and Ziemba (2004), Zha (2007), Adam et al. (2008), Huang and Milevsky (2008), Pelizzon and Weber (2009). Although portfolio selection is of major concern to portfolio managers it has taken some time until there were attempts to include derivatives into dynamic portfolio optimization. This might be partially explained by the fact that initially, derivatives were seen as redundant securities which can be replicated by implementing a dynamic trading strategy in stocks and bonds. The standard Black/Scholes option pricing model supports this view. However, more advanced option pricing models take additional risk factors such as stochastic volatility into account. In these models, markets are incomplete and derivatives are no longer replicable by stocks and bonds alone. Instead they provide opportunities to earn additional risk premiums. Liu and Pan (2003), Branger et al. (2008) address this idea and develop models for single stock economies and analyze implications of derivatives on portfolio management in the presence of stochastic jumps and volatility.

In practice several asset classes and risk factors are relevant for portfolio choice though. For instance Fama and French (1993) show that three factors are necessary to reflect that small stocks and stocks with high book-to-market-ratio tend to earn additional returns. Another example from the context of international portfolio selection are emerging markets. Several studies including e.g. Bekaert and Urias (1999), Bekaert et al. (1998), De Santis and Imrohoroglu (1997) suggest that emerging markets yield high returns and low correlations to stock markets of industrialized countries indicating potential benefits from considering them for international portfolio investment.

This paper contributes to the literature in several aspects. First, we solve the portfolio-planning problem in a jump-diffusion model when there are several stocks and the market is complete ("full access to the derivatives market"). Second, we analyze the case when the investor has access to some derivatives but still faces an incomplete market ("partial access to the derivatives market"). Third, in a numerical application, we study utility gains that can be obtained by successively introducing one derivative after the other. This leads to implications concerning (a) the number of derivatives needed and (b) how these derivatives should look like. For both full and partial access to the derivatives market we determine optimal factor exposures as the solution to a system of ordinary differential equations. In the presence of jump risk the numerical analysis highlights that the utility gain due to introducing yet another derivative can be increasing or decreasing in the number of derivatives which are already traded if the derivatives to be introduced and their characteristics are pre-specified. Potential utility gains follow from a complex relationship of jump probabilities, risk

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aversion, and the structure of risk factors available. Intuitively, the latter can be explained by the matter of fact that derivatives make more attractive "packages" of risk factors feasible. The amount by which an investor profits from the introduction of a new standard derivative depends on how suboptimal the current relation between exposures to risk factors is. As such even if market prices of risk are constant the utility gain from a new derivative depends on diffusion correlation and volatility as well as jump sizes. In extreme cases the introduction of a new derivative does not lead to an improvement at all. Depending on the packages of risk factors available, investors have different preferences for derivatives which allow for exposure optimization of individual risk factors. This is highlighted when derivatives are considered which provide an exposure to a single risk factor only. For instance, we analyze insurance contracts which make protection with respect to individual jump-risk factors tradable. Our numerical results indicate that there might be situations in which investors benefit more from trading insurance contracts on and thus sensitivities to a particular jump factor than derivatives which cover several or even all other risk factors combined. Hence, if an investor could decide on which type of derivative to introduce next he would always chose a contract which provides exposure to a the risk factor he wants to trade most. In this case, utility gains would become a decreasing function in the number of derivatives. These considerations might also be taken into account by futures and options exchanges or other financial intermediaries when developing new financial products/derivatives in order to attract the interest of portfolio managers.

The remainder of the paper is structured as follows. Section 2 establishes the general model and the approach to asset allocation. Section 3 deals with full access to the derivatives market. Section 4 addresses the question of partial access in the presence of jumps. Numerical applications are discussed in Section 5. Finally, Section 6 concludes.

2. General model

We begin with a general model for portfolio choice in continuous time and discuss special cases in subsequent sections. More precisely, we assume that in the economy a finite number I of stocks are traded at prices $S \in \mathbb{R}^I$ which are influenced by diffusion and jump factors. The market is frictionless, i.e. there are no transaction costs, short selling is allowed, and stocks are perfectly divisible. Investors can both borrow and lend money at the risk-free interest rate R. Additionally, L stochastic state variables $X \in \mathbb{R}^L$ are observed. Under the data generating measure $\mathscr{P}$ the market can be described by the following stochastic processes

$$
\begin{aligned}
dS_i &= S_i\left\{R + \sum_{j=1}^{J} a_{i,j}V_j\eta_j + \sum_{k=1}^{K}\mu_{i,k}\left[v^{\mathscr{P}}h_k^{\mathscr{P}} - v^{\mathscr{Q}}h_k^{\mathscr{Q}}\right]\right\}dt \\
&\quad + S_i\sum_{j=1}^{J} a_{i,j}\sqrt{V_j}dW_j^{\mathscr{P}} + S_i\sum_{k=1}^{K}\mu_{i,k}\left(dN_k - v^{\mathscr{P}}h_k^{\mathscr{P}}dt\right), \\
dX_l &= m_l^{\mathscr{P}}dt + \sum_{j=1}^{J+L} b_{l,j}\sqrt{V_j}dW_j^{\mathscr{P}} + \sum_{k=1}^{K}\mu_{I+l,k}dN_k, \\
V_j &= \bar\beta_j + \beta_j \cdot X, \\
v^{\mathscr{P}} &= \bar\lambda^{\mathscr{P}} + \lambda^{\mathscr{P}} \cdot V, \\
v^{\mathscr{Q}} &= \bar\lambda^{\mathscr{Q}} + \lambda^{\mathscr{Q}} \cdot V, \\
R &= \bar\rho + \rho \cdot X, \\
m_l^{\mathscr{P}} &= \bar\varepsilon_l^{\mathscr{P}} + \varepsilon_l^{\mathscr{P}} \cdot X,
\end{aligned} \tag{1}
$$

where $J, K \in \mathbb{N}, V \in \mathbb{R}^{J+L}, v^{\mathscr{P}}, v^{\mathscr{Q}} \in \mathbb{R}$, and $m^{\mathscr{P}} \in \mathbb{R}^L$. To simplify notation we drop time subscripts to stochastic processes or to state variables. As usual V has the interpretation of a vector of stochastic factor variances. Furthermore, $a \in \mathbb{R}^{I\times J}, b \in \mathbb{R}^{L\times(J+L)}, \beta \in \mathbb{R}^{(J+L)\times L}, \rho \in \mathbb{R}^L, \varepsilon^{\mathscr{P}} \in \mathbb{R}^{L\times L}, \lambda^{\mathscr{P}}, \lambda^{\mathscr{Q}} \in \mathbb{R}^{J+L}, h^{\mathscr{P}}, h^{\mathscr{Q}} \in \mathbb{R}^K, \eta \in \mathbb{R}^{J+L}, \bar\beta \in \mathbb{R}^{J+L}, \bar\varepsilon^{\mathscr{P}} \in \mathbb{R}^L, \mu \in \mathbb{R}^{(I+L)\times K}$ as well as $\bar\rho, \bar\lambda^{\mathscr{P}}, \bar\lambda^{\mathscr{Q}} \in \mathbb{R}$ are deterministic, probably time dependent. Stochastic shocks are introduced by $W^{\mathscr{P}}$ which is a multidimensional standard Wiener process on $\mathbb{R}^{J+L}$ and the multivariate point process N on $\mathbb{R}^K$. For some quantities we add the superscript $\mathscr{P}$ to indicate that they might change when switching to a risk-neutral probability measure $\mathscr{Q}$ yet to be defined. The point process N_k jumps with intensity $v^{\mathscr{P}}h_k$, i.e. the probability of a jump over a small time interval Δt is approximately $v^{\mathscr{P}}h_k\Delta t$. If we require that $\sum_{k=1}^{K}h_k = 1$ then h_k is the probability that point process N_k jumps conditional on a jump occurring. Furthermore, $v^{\mathscr{Q}}$ and $h^{\mathscr{Q}}$ correspond to $v^{\mathscr{P}}$ and $h^{\mathscr{P}}$ under the risk neutral measure $\mathscr{Q}$.

The stock prices are sensitive to the first J diffusion factors.[1] Furthermore, we observe L state variables which are exposed to the additional risk factors. There is thus diffusion risk which is not spanned by the stocks if $L > 0$. The state variables may characterize e.g. factor variances $V_j, j \in \{1, \ldots, J+L\}$, or the level of the short term interest rate R. Note that both R and V_j are linear functions in the state variables. Furthermore, jump intensities might depend linearly on local variances and thus state variables as well. The multivariate point process may affect stocks and state variables simultaneously. The sensitivities of stocks and state variables are given by the jump sizes $\mu_{i,k}, i \in \{1, \ldots, I+L\}, k \in \{1, \ldots, K\}$. If $\mu_{i,k} \neq 0$ the stock or state variable is sensitive to a point process N_k. Thus, the choice of this exposure allows to model jumps that have an impact on stocks only, on state variables only, and on both. The model nests several models which are similar or identical to models with stochastic volatility and jumps like those of Heston (1993), Merton (1976), Bakshi et al. (1997), Bates (1996, 2000).[2]

We assume that technical conditions are met such that the model (1) is well defined.[3] More precisely, we make the following assumption.

Assumption 1. All processes considered are sufficiently well-behaved and there are no redundant assets or state variables.

The market is incomplete to the diffusion factors $W_{J+1}^{\mathscr{P}}, \ldots, W_{J+L}^{\mathscr{P}}$. Furthermore, if $I \leqslant J + K$ then not all jump-risk factors are spanned by the stocks. In these situations the introduction of derivative securities might complete the market.[4] Consider for instance the case of the Heston model. It allows for stochastic stock prices and variances with arbitrary correlation ϱ. This model is a special case of (1) and given by

$$
\begin{aligned}
dS &= S\{R + X\eta_1\}dt + S\sqrt{X}dW_1^{\mathscr{P}}, \\
dX &= \kappa^{\mathscr{P}}(\overline{V} - X)dt + \varrho\bar\sigma\sqrt{X}dW_1^{\mathscr{P}} + \sqrt{1-\varrho^2}\bar\sigma\sqrt{X}dW_2^{\mathscr{P}},
\end{aligned} \tag{2}
$$

where $V_1 = X$. The stock price is sensitive to the first Wiener process $W_1^{\mathscr{P}}$ but provides no exposure to $W_2^{\mathscr{P}}$. Only the variance $V_1 = X$ is linked to $W_2^{\mathscr{P}}$. Since the price of a derivative security depends on the observed variance trading in such a contract enables the investor to generate arbitrary exposures to both diffusion factors. The market is complete and as a consequence investors can replicate any additional claim by trading in the stock, the bond, and the first derivative security. This example illustrates the reason for considering derivative securities in portfolio strategies in an

[1] Note that it might be that $I \neq J$. This allows to capture e.g. the two-factor stochastic volatility model of Bates (2000) as special case.

[2] Note that Merton (1976), Bakshi et al. (1997), Bates (1996, 2000) assume continuous jump distributions. Therefore, the model discussed in this paper cannot match them exactly.

[3] For instance, we must ensure that variances V_j cannot become negative. See Duffie and Kan (1996) for details in the context of pure diffusion models.

[4] If the market is not complete then the investor might focus on a subset of jump and diffusion factors only and thus use them for portfolio-planning. A detailed analysis is presented in Section 4 when the factor subsets $\mathbb{M}$ and $\mathbb{K}$ are introduced.