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Optimal dividend and equity issuance problem with proportional and fixed transaction costs

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ABSTRACT

This paper investigates the optimal dividend problem of an insurance company, which controls risk exposure by reinsurance and by issuing new equity to protect from bankruptcy. Transaction costs are incurred by these business activities: reinsurance is non-cheap, dividend is taxed and fixed costs are generated by equity issuance. The goal of the company is to maximize the expected cumulative discounted dividend minus the expected discounted costs of equity issuance. This problem is formulated as a mixed regular-singular-impulse stochastic control problem. By solving the corresponding HJB equation, we obtain the analytical solutions of the optimal return function and the optimal strategy.

1. Introduction

Optimal dividend and reinsurance problems have attracted much attention in recent years. Two classical papers by Jeanblanc-Picqué and Shiryaev (1995) and Asmussen and Taksar (1997) studied the optimal dividend problem for an insurance company whose surplus process follows Brownian motion with drift. Højgaard and Taksar (1999) and Asmussen et al. (2000) explored the proportional reinsurance and excess-of-loss reinsurance respectively to maximize the expected present value of dividend in a diffusion approximation model. Choulli et al. (2001, 2003) investigated the optimal reinsurance and optimal dividend problems for a company with debt liability and constraints on risk reduction. When there were transaction costs for each dividend payment, Cadenillas et al. (2006) and Bai et al. (2010) studied the optimal dividend problem with proportional and excess-of-loss reinsurance, respectively.

However, in most of these papers, ruin happens for an insurance company with probability 1. This is not practically interesting. It may be required by some Regulatory Authority that the insolvency company should issue equities or inject capital to protect the interests of policyholders. This idea may go back to Borch (1974, Chap. 20) and Harrison and Taylor (1978), and recent references on capital injections including Avram et al. (2007) for spectrally negative Lévy processes, Løkka and Zervos (2008), He and Liang (2009) and Meng and Siu (2011) for the diffusion model, Yao et al. (2011) for the dual model, Dai et al. (2010) and Avanzi et al. (2011) for the dual model with diffusion, and Kulenko and Schmidli (2008) and Scheer and Schmidli (2011) for the Cramér–Lundberg model. A detailed discussion of this topic on minimizing the expected discounted capital injections can be found in Eisenberg (2010).

In this paper, we study the optimal dividend problem for an insurance company, which controls risk exposure by reinsurance and by issuing new equity to protect from bankruptcy. We assume all these business activities will incur transaction costs: reinsurance company will demand more premium for the risk ceded by the insurer; dividends will be taxed; fixed costs are generated by advisory and consulting fees when issuing new equities. This leads to a mixed regular-singular-impulse stochastic control problem. Our objective is to find out the value function and the optimal policies which maximize the expected cumulative discounted dividend minus the expected discounted costs of equity issuance. Yao et al. (2011) considered the problem in the dual model for pharmaceutical or petroleum companies. Meng and Siu (2011) studied the optimal mixed impulse-equity insurance

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control problem with excess-of-loss reinsurance, but fixed costs were incurred by dividend payout, not by equity issuance as in this paper. So the methods given by Taksar (2000) and employed by Meng and Siu (2011) cannot be applied by our model. He and Liang (2009) investigated the optimal financing and dividend control problem with fixed and proportional transaction costs. We consider non-cheap reinsurance in this paper, this makes it more difficult to solve the corresponding HJB equation. In addition, we prove a result in Section 3 which states the optimal timing of equity issuance is when the surplus process hits barrier 0.

The outline of this paper is as follows. In the next two sections, we first give a rigorous mathematical formulation of the problem, and then analyze some properties of the value function in Section 3. We give the HJB equation of value function and prove the verification theorem in Section 4. Section 5 is devoted to the derivation of the solution of the model without reinsurance, which is interesting in its own right and will provide some insights into the resolution of the general problem. In Section 6, we solve the HJB equation in the general model with reinsurance, and give two numerical examples to illustrate the impacts of the transaction costs on the optimal strategy.

2. Problem formulation

Let $(\Omega, \mathcal{F}, P)$ be a probability space with the filtration $\{\mathcal{F}_t\}$ satisfying the usual conditions. Our results will be formulated within the framework of the controlled diffusion model. However, for the purpose of motivation it is convenient to start from the classical Cramér–Lundberg model, in which the reserve of the insurance company is modeled as

$$R_t = x + pt - \sum_{i=1}^{N_t} X_i, \tag{2.1}$$

where $x \geq 0$ is the initial reserve, $\{N_t\}$ is a Poisson process with intensity $\beta > 0$, and the individual claim sizes $X_1, X_2, \ldots,$ independent of $\{N_t\}$, are i.i.d positive random variables with finite first and second moments μ_1, μ_2. The premium rate p is calculated by the expected premium principle,

$$p = (1+\mu)\beta\mu_1, \tag{2.2}$$

where $\mu > 0$ is the relative safety loading of the insurer.

We now consider proportional reinsurance for the above classical model. Let $a \in [0, 1]$ denote the (fixed) proportional retention level, the reinsurance company demands premiums at the rate of $(1+\lambda)\beta\mu_1(1-a)$, where $\lambda \geq \mu$. When $\lambda = \mu$ the reinsurance is *cheap*, otherwise it is *non-cheap*. Then the reserve process is given by

$$R_t^a = x + p^a t - \sum_{i=1}^{N_t} aX_i, \tag{2.3}$$

where $p^a = \beta\mu_1(\mu - \lambda + a(1+\lambda))$. According to Grandell (1979), the diffusion approximation is described by

$$R_t^a = x + \beta\mu_1 \int_0^t (a\lambda - (\lambda - \mu))\, dt + a\sqrt{\beta\mu_2} W_t,$$

where $\{W_t\}$ is a standard Brownian motion, adapted to the filtration $\{\mathcal{F}_t^W \triangleq \sigma(W_s; 0 \leq s \leq t)\}$. Augmenting this filtration with the collection of P-null sets, with slight abuse of notation we still write as $\{\mathcal{F}_t\}$. Assume the insurer can dynamically choose the retention level, for notational convenience we consider a model of the form

$$R_t^a = x + \int_0^t (a_s\lambda - (\lambda - \mu))\, ds + \sigma \int_0^t a_s\, dW_s, \tag{2.4}$$

which results from replacing $a, \lambda\beta\mu_1, \mu\beta\mu_1$ and $\sqrt{\beta\mu_2}$ by a_s, λ, μ and σ, respectively.

In addition, we incorporate dividend payment and equity issuance into the above model. Let $\{L_t\}$ denote the total amount of dividend paid until time t. The equity issuance process $\{G_t\}$ is described by a sequence of increasing stopping times $\{\tau_n | n = 1, 2, \ldots\}$ and a sequence of random variables $\{\xi_n | n = 1, 2, \ldots\}$, which are associated with the timings and the amounts of equity issuance. With a *control strategy* $\pi = \{a^\pi; L^\pi; G^\pi\} = \{a^\pi; L^\pi; \tau_1^\pi, \ldots \tau_n^\pi, \ldots; \xi_1^\pi, \ldots, \xi_n^\pi, \ldots\}$, the dynamics of the controlled surplus process $\{R_t^\pi\}$ are given by

$$R_t^\pi = x + \int_0^t (a_s^\pi \lambda - (\lambda - \mu))\, ds + \sigma \int_0^t a_s^\pi\, dW_s + \sum_{n=1}^{\infty} I_{(\tau_n^\pi \leq t)} \xi_n^\pi - L_t^\pi. \tag{2.5}$$

We have the following definition of an admissible strategy that can be selected by the insurer.

Definition 2.1. A strategy π is said to be *admissible* if

(i) $\{a_t^\pi\}$ is an $\{\mathcal{F}_t\}_{t\geq 0}$-adapted process and $a_t^\pi \in [0, 1]$ for any $t \geq 0$.
(ii) $\{L_t^\pi\}$ is an increasing, $\{\mathcal{F}_t\}_{t\geq 0}$-adapted cádlág process, and $\triangle L_t^\pi \leq R_{t-}^\pi$.
(iii) τ_n^π is a stopping time with respect to $\{\mathcal{F}_t\}_{t\geq 0}$ and $0 \leq \tau_1^\pi < \tau_2^\pi < \cdots < \tau_n^\pi < \cdots a.s.$
(iv) ξ_n^π is nonnegative and measurable with respect to $\mathcal{F}_{\tau_n^\pi}$, $n = 1, 2 \ldots.$
(v) $P(\lim_{n\to\infty} \tau_n^\pi \leq T) = 0, \forall\, T \geq 0$.
(vi) $R_t^\pi \geq 0$, $a.s.$

Condition (ii) implies that the amount of dividend payout is no more than the reserve available at that moment. Condition (v) means that the equity issuance may not occur infinitely in a finite time interval. Condition (vi) demands that the insurance company should not go bankrupt. This reflects the fact that the insurance company is under regulation by some authorities to protect the interests of policyholders.

Denote the set of all admissible strategies with initial value x by Π_x. For each admissible strategy π, we define the performance function as

$$V(x, \pi) = \limsup_{t\to\infty} E^x\left[\int_0^t e^{-rs}\beta_1\, dL_s^\pi - \sum_{n=1}^{\infty} e^{-r\tau_n^\pi}(K + \xi_n^\pi \beta_2) I_{\{\tau_n^\pi \leq t\}}\right]. \tag{2.6}$$

This is the expected discounted present value of the dividend payout minus the equity issuance in the infinite time horizon. Here $\beta_1 < 1$ in the dividend payout process implies the proportional transaction costs generated by the tax, and $(\beta_2 - 1)\xi$ and K are proportional costs and fixed costs to meet the equity issuance of amount ξ, where $\beta_2 > 1$.

The objective is to find the optimal return function, or the *value function*, defined as

$$V(x) = \sup_{\pi \in \Pi_x} V(x, \pi) \tag{2.7}$$

and the optimal strategy π^* such that $V(x) = V(x, \pi^*)$.

Remark 2.1. Another definition of admissible strategy adopted by Løkka and Zervos (2008) and He and Liang (2009) allows the company to liquidate assets and get out of the business, i.e., condition (vi) is not required. Furthermore the company aims at maximizing the expected discounted dividend payments minus the expected discounted costs of equity issuance before the surplus process falls below 0. Under such considerations, we denote by