



A closed-form approximation for the fractional Black–Scholes model with transaction costs



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ABSTRACT

In this paper, we investigate option valuation problems under the fractional Black–Scholes model. The aim is to propose a pricing formula for the European option with transaction costs, where the costs structure contains fixed costs, a cost proportional to the volume traded, and a cost proportional to the value traded. Precisely, we provide an approximate solution of the nonlinear Hoggard–Whalley–Wilmott equation. The comparison results reveal that our approximate solutions are close to the numerical computations. Moreover, the comparison results demonstrate that the price of the European option decreases as the Hurst exponent increases.

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1. Introduction

The fractional Brownian motion (fBm) was first introduced by Kolmogorov in 1940 in [1]. Mandelbrot and Van Ness provided a stochastic integral representation of this process in terms of a standard Brownian motion in 1968 in [2]. The name fBm is due to the stochastic integral representation in terms of a standard Brownian motion. To capture the property of long-range dependence in financial market series [3], the fractional Black–Scholes model replaces the Brownian motion in the Black–Scholes (BS) model [4] with the fBm. Since the fBm is not a semimartingale, the arbitrage opportunities exist in the fractional Black–Scholes model under a complete and frictionless setting. A considerable number of arbitrage strategies for fBm models is provided by Rogers [5], Shiryayev [6], Salopek [7] and Cheridito [8]. Besides the financial problems, the fractional calculus is also applied to other real problems [9,10].

In the complete and frictionless market, Black and Scholes [4] constructed a self-finance trading strategy that replicates the option's value continuously. However, the replicating portfolio requires continuous trading and, in the market with transaction costs, the continuous trading policy will incur an infinite amount of trading cost. Therefore, the BS replicating strategy is no longer valid. Barles and Soner [11] pointed out that in such a market, there is no portfolio that replicates the final payoff of a European option.

Leland [12] assumed that the hedging takes place at the given discrete intervals and derives an option pricing formula. Furthermore, under the assumption of the discrete time hedge policy, Hoggard et al. [13] relaxed the assumption of the convexity of the resulting option price and derived a nonlinear BS equation by extending the BS equation to incorporate the transaction costs. The transaction costs in [13,12] are only proportional to the value traded. However, the transaction costs are also proportional to the volume traded or the fixed costs. The extended Leland model in [14] plays an important role in option pricing with the transaction costs.

Guasoni [15] showed that the arbitrage opportunities are excluded when the proportional transaction costs are included in the fractional Black–Scholes model. Under Guasoni's no-arbitrage criterion, the Leland transaction cost model is extended

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to the fractional Black–Scholes model with transaction costs [16–18,3,19]. Precisely, they derived a nonlinear partial differential equation for pricing the European option or other derivatives. The solution of such a nonlinear equation is obtained numerically. To date, there have been no published results that propose the closed form or the approximate solution for the extended Leland model.

In this paper, we provide an approximate solution of the fractional pricing equation with transaction costs by using the variational iteration method (VIM). As we only consider the fixed costs, the exact solution is obtained directly by using the VIM. The VIM [20] has been widely applied to analyze the nonlinear boundary value problems [21], the nonlinear heat diffusion equations [22] and the nonlinear reaction–diffusion equations [23]. In recent years, the VIM is applied to the nonlinear real problems [24]. To date, there have been no publications on the application of the VIM to solve nonlinear financial problems. The comparison results reveal that our approximation is very accurate. Moreover, we demonstrate that the price of the European option decreases as the Hurst exponent increases. Besides the VIM, many methods, such as the homotopy perturbation method [25,26] and the Laplace transform [27], can be applied to find the approximate solution of the nonlinear financial problems.

This paper is organized as follows. In Section 2, we introduce the fractional Black–Scholes model and derive the fractional pricing equation with transaction costs. In Section 3, we propose an approximate formula of a European option in the fractional Black–Scholes model with transaction costs. In Section 4, we provide a comparison result to demonstrate the accuracy of our approximate formula. Finally, a concise conclusion is provided in Section 5.

2. A fractional Black–Scholes valuation model with transaction costs

Wang et al. [18] pointed out that traders either underreact or continually overreact to news since they are bonded rational. The behavior may lead to the features of “leptokurtic” and long range dependence in stock return time series. The fBm may be a useful tool for capturing this phenomenon.

Definition 2.1. Let $(\Omega, F, (\mathcal{F}_t)_{0 \leq t \leq T}, P)$ be a complete probability space. A fractional Brownian motion (fBm) B_t^H with Hurst exponent $H \in (0, 1]$ is continuous, centered Gaussian processes with covariance functions

$$\text{cov}(B_t^H, B_s^H) = \frac{1}{2}(|t|^{2H} + |s|^{2H} - |t - s|^{2H}), \quad s, t \in \mathbb{R}. \quad (1)$$

As $H = \frac{1}{2}$, B^H is a standard Brownian motion. For $H \in (\frac{1}{2}, 1]$ the correlation of two increments of B^H over non-overlapping time intervals is positive, and for $H \in (0, \frac{1}{2})$ it is negative. From (1), we also have $E[(B_t^H - B_s^H)^2] = |t - s|^{2H}$. Moreover, fBm has a stationary increment and is self-similar with parameter H , that is the Hurst parameter H is also the self-similarity parameter.

The fractional Black–Scholes model is given as

$$S_t = S_0 \exp(rt + \sigma B_t^H), \quad t \in [0, T],$$

where S_0, σ are positive constants and r is a real constant. In the sense of a path integral, we have

$$dS_t = rS_t dt + \sigma S_t dB_t^H.$$

For $H \neq \frac{1}{2}$, (B_t^H) is not a semimartingale. This implies that in the fractional Black–Scholes model there exists a weak form of arbitrage called “free lunch with vanishing risk” from Theorem 7.2 of Delbaen and Schachermayer [28]. So far a considerable number of arbitrage strategies for fBm models is provided by Rogers [5], Shiryaev [6], Salopek [7] and Cheridito [8].

In the fractional Black–Scholes model, Cheridito [8] showed that the arbitrage probabilities can be excluded by suitably restricting the class of allowed trading strategies. Moreover, Guasoni [15] established a no-arbitrage criterion that reduces the arbitrage opportunities under propositional transaction costs to the condition that the asset price process move arbitrarily little over arbitrarily large time intervals. Under Guasoni’s no-arbitrage criterion, Wang et al. [18], Jumarie [16] and Xiao et al. [19] extended the Leland transaction cost model to the fractional Black–Scholes model with transaction costs. Precisely, they derived a nonlinear partial differential equation for pricing the European option or other derivatives. The solution of such a nonlinear equation is obtained numerically.

In this paper, we extend Hoggard’s transaction cost model to the fractional Black–Scholes model. Moreover, the cost structure not only contains a cost proportional to the value traded but also contains fixed costs and a cost proportional to the volume traded. Let α_1, α_2 , and α_3 be the fixed cost component, the cost proportional to the volume traded, and the cost proportional to the value traded, respectively. The cost structure $R(S, |v|)$ is given as

$$R(S, v) = \alpha_1 + (\alpha_2 + \alpha_3 S)|v|.$$

We find that the expected transaction costs are to leading order

$$E \left[R \left(S, \left| \frac{\partial^2 V}{\partial S^2} \right| \right) \right] = \frac{\alpha_1}{\Delta t} + \sigma \left(\frac{\alpha_2}{S} + \alpha_3 \right) \sqrt{\frac{2}{\pi}} S^2 \left| \frac{\partial^2 V}{\partial S^2} \right| (\Delta t)^{H-1} \equiv F \left(S, \left| \frac{\partial^2 V}{\partial S^2} \right| \right)$$

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