



Optimal inventory policy with fixed and proportional transaction costs under a risk constraint

S.Y. Wang^a, K.F.C. Yiu^{b,*}, K.L. Mak^c

^a School of Economic Management, North College of Beijing University of Chemical Technology, Yanjiao, East Beijing, PR China

^b Department of Applied Mathematics, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China

^c Department of Industrial and Manufacturing Systems Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong, China

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ABSTRACT

The traditional inventory models focus on characterizing replenishment policies in order to maximize the total expected profit or to minimize the expected total cost over a planned horizon. However, for many companies, total inventory costs could be accounting for a fairly large amount of invested capital. In particular, raw material inventories should be viewed as a type of invested asset for a manufacturer with suitable risk control. This paper is intended to provide this perspective on inventory management that treats inventory problems within a wider context of financial risk management. In view of this, the optimal inventory problem under a VaR constraint is studied. The financial portfolio theory has been used to model the dynamics of inventories. A diverse portfolio consists of raw material inventories, which involve market risk because of price fluctuations as well as a risk-free bank account. The value-at-risk measure is applied thereto to control the inventory portfolio's risk. The objective function is to maximize the utility of total portfolio value. In this model, the ordering cost is assumed to be fixed and the selling cost is proportional to the value. The inventory control problem is thus formulated as a continuous stochastic optimal control problem with fixed and proportional transaction costs under a continuous value-at-risk (VaR) constraint. The optimal inventory policies are derived by using stochastic optimal control theory and the optimal inventory level is reviewed and adjusted continuously. A numerical algorithm is proposed and the results illustrate how the raw material price, inventory level and VaR constraint are interrelated.

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1. Introduction

Inventories are stocks of raw materials, components and finished goods that are stored in warehouses, the instrumentalities of transportation, and retail stores. Raw material inventories are necessary to manufacturers because they create buffers against irregular supplies and demand shifts, guarantying product availability. Yet, according to Ballou [1], stockpiling inventory may result in costs in the range of 20%–40% of annual invested capital. Thus, good inventory control will provide lower costs and promote overall company performance. Recognizing the importance of inventory management, copious literature investigating optimal inventory strategies exists. Arrow et al. [2] laid the foundation of modern inventory theory, in which expected costs are chosen as an objective function. The traditional inventory models focus on characterizing replenishment policies in order to maximize the total expected profit or to minimize the expected total cost over a planned horizon. Examples include the popular (EOQ) model and the (s, S) model [3,1,4]. The conventional inventory control methods are appropriate for risk-neutral managers, but in reality most managers are risk averse [5], and in the field of the supply chain,

* Corresponding author. Tel.: +852 27666923; fax: +852 2362 9045.

E-mail address: macyiu@polyu.edu.hk (K.F.C. Yiu).

risk analysis and risk control have become more and more important [6,7]. Therefore, techniques that consider both risks and returns of holding inventory are crucial.

Risk management is increasingly important in many disciplines, ranging from banking [8] and technology [9–12] to humanity and ecology [13,14]. The literature on inventory models with risk control is quite limited and is mainly focused on discrete-time problems. Eeckhoudt et al. [15] investigate the effects of risk and risk aversion in the single-period inventory (“newsvendor”) problem Agrawal and Seshadri [16] consider the single-period inventory problem for a risk-averse retailer, whose objective is to maximize expected utility. Bouakiz and Sobel [17] optimize the multi-period news vendor model with an exponential utility criterion. Chen et al. [18] extend the multi-period inventory model with a general risk-averse utility function and illustrate with numerical results the effects of risk aversion on inventory strategies. In recent years, the VaR measure has also received attention in the supply chain [19]. Some literatures of inventory management have also used VaR as a risk control measure. Luciano et al. [20] consider a standard multi-period static inventory model and define the optimal policy as one that maximizes the expected discounted profits. The mean and variance of profits (and costs) are obtained for both finite and infinite cycles. The VaR induced by the optimal replenishment policy is estimated by applying probability theory Tapiero [21] attempts to formulate a single period inventory model which is to minimize the VaR of the total cost. So far, the models considered are discrete in nature with fixed cycle length between replenishments. Clearly, sophisticated stochastic models have not been used to model inventory price fluctuation and continuous replenishment policy has not been studied.

The recent violent fluctuations in commodity prices have forced many manufacturers to endure large market risk. Under these circumstances material inventories not only ensure production availability, but are also a kind of investment asset. Their price fluctuation nature closely resembles the risky assets of a financial portfolio. Undoubtedly, the concept of portfolio theory has been well developed in finance. In this paper, the financial portfolio theory will be applied to address the inventory control problem.

The specific focus herein will be the industrial manufacturer’s raw material inventory. It is assumed that the manufacturer can sell the raw material inventories back to the suppliers at a discounted market price. And once a non-zero order is placed, the manufacturer has to pay a fixed ordering cost to the supplier except purchasing costs. The problems are then addressed from the perspective of financial portfolio theory with fixed and proportional transaction costs. The raw material inventory required by a manufacturer is considered as an investment, and a portfolio consisting of these material inventories as well as a risk free bank account is explored. To exercise proper risk control over the inventory portfolio value, the VaR constraint is imposed continuously over time. The objective function is to maximize the total expected utility of the portfolio value during the horizon. The optimal ordering and selling conditions are derived by using stochastic optimal control theory. Under this formulation, the optimal inventory level can be reviewed and adjusted continuously. By applying the VaR constraint, and assuming that portfolio allocations do not change over a short horizon period, we indicate that holding value in the raw material inventory is reduced whenever the VaR constraint becomes active.

The rest of the paper is organized as follows. In Section 2, the model for the continuous-time optimal inventory portfolio without VaR constraint is derived for one raw material inventory plus a risk-free asset. After that, the VaR constraint is imposed in Section 3 and the optimal ordering and selling conditions are derived. The final optimal inventory policy under a VaR constraint is continuously reviewed and adjusted. Finally, in Section 4, a numerical algorithm is proposed to solve the optimal control problem and illustration examples are presented in Section 5.

2. Inventory model without VaR constraint

2.1. Model formulation

Consider an industrial plant producing only one kind of product and requiring only one type of raw material. Our focus is on the inventory policies of the raw material. Because of fluctuations in the price of the raw material, holding onto an inventory is a kind of risky asset. Given a sufficient amount of capital to maneuver, the manager has two choices: to invest in a risk free asset, such as bank notes, or to maintain raw material inventories. From the perspective of finance, the manager has a portfolio consisting of one risk free asset and one risky asset. For simplicity, the following assumptions are made:

- In the planned time horizon, the price of product is a deterministic function in respect to time t , which is exogenously determined by markets.
- The demand of product and the price of raw material are independent to each other.
- There is no lead time for the raw material.

First, some notations are introduced:

- $\omega_0(t)$ is the risk free asset value at time t .
- $\omega_1(t)$ is the holding value of raw material inventory at time t .
- $X(t)$ is the total value of the risk free asset plus the raw material inventory, that is

$$X(t) = \omega_0(t) + \omega_1(t).$$

- $S_0(t)$ is the risk free asset price at time t .
- $S_1(t)$ is the raw material price at time t .

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