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Pricing currency options with support vector regression and stochastic volatility model with jumps

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ABSTRACT

This paper presents an efficient currency option pricing model based on support vector regression (SVR). This model focuses on selection of input variables of SVR. We apply stochastic volatility model with jumps to SVR in order to account for sudden big changes in exchange rate volatility. We use forward exchange rate as the input variable of SVR, since forward exchange rate takes interest rates of a basket of currencies into account. Therefore, the inputs of SVR will include moneyness (spot rate/strike price), forward exchange rate, volatility of the spot rate, domestic risk-free simple interest rate, and the time to maturity. Extensive experimental studies demonstrate the ability of new model to improve forecast accuracy.

1. Introduction

The foreign exchange market is the deepest, largest and most liquid financial market in the world. There has been a thriving over-the-counter market for currency options, which reflects the growing need to manage exchange rate risk in the integrated global economy. And appropriate and accurate option pricing is crucial for proper use of currency options.

Option pricing was revolutionized by Black–Scholes in 1973. However, a lot of evidence indicates that option pricing models premised upon Black–Scholes model exhibiting severe error when fitted to market data. In 1983 Garman and Kohlhagen extended the Black–Scholes model to cope with the presence of two interest rates. Many methodologies for the currency options pricing have been proposed by using the modification of Garman–Kohlhagen (GK) model, such as Amin and Jarrow (1991), Heston (1993), Ekvall (1997), Rosenberg (1998) and Bollen and Rasiel (2003).

In those parametric models it is quite difficult to justify selection of one parametric specification over the other. This leads to serious problem of misspecification. Market participants change their option pricing attitudes from time to time, a stationary nonlinear relationship between theoretical option prices and many variables described by parametric models may fail to adjust to such rapidly changing market behavior. In recent years, many scholars have also turned to nonparametric methods. Artificial neural network (ANN) is introduced into option pricing by Hutchison in 1994. Neural networks confirm their usefulness in modeling option pricing due to their data-driven and nonparametric weak properties. Then a lot of machine learning techniques are used for calculating the options value. Support vector machine proposed by Vapnik (1995) is another hot topic in machine learning following neural network. Support vector machine is developed on the basis of statistical learning theory. It is approximate implementation of structural risk minimization (SRM) induction principle that aims at minimizing a bound on the generalization error of a model, rather than minimizing only the mean square error over the data set. Support vector machines are used for classification and regression (SVR) (Vapnik, Golowich, & Smola, 1997). SVR is a powerful machine learning method that is useful for constructing data-driven nonlinear empirical process models. It shares many features with neural networks but possesses some additional desirable characteristics. SVR overcomes one of important weaknesses of neural networks that they cannot avoid to get trapped in local minima (Gunn, Brown, & Bossley, 1997). SVR is gaining widespread acceptance in data-driven nonlinear modeling applications (Cao, 2003; Fan & Palaniswami, 2000). In recent years, some scholars have also turned to SVR method for option pricing (Kim, 2003; Xun, Zhang, Xiao, & Chen, 2009). Xun (2009) focus on capturing error residuals. Instead of researching into complicated option market directly, SVR is implemented based on results of three well known and successful traditional methods in Xun model (2009).

Most authors focus on architectures incorporating nonparametric methods for option pricing. However the selection of reasonable input variables is also important. According to Garman–Kohlhagen formula, following notations are used: underlying asset price, option's exercise price, asset price volatility, domestic current risk-free interest rate, foreign current risk-free interest rate, and expiration time. This study has two goals for selection of input variables based on the existing literatures.

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The first goal is to use forward exchange rate as the input variable of SVR to develop a new currency options pricing model. The behaviors of the spot exchange rate and two economic interest rates must be linked in order to prevent arbitrage chances. And forward exchange rate reflects the expectation of future spot rate. Using the forward exchange rate as the input variable of SVR can take interest rates of the pair of currencies into account.

The second goal is to estimate exchange rate volatility with stochastic volatility model with jumps (SVJ). The Other inputs can be obtained directly from market data except for currency volatility. SV models are used to model the volatility of foreign exchange rates (Harvey & Shephard, 1994; Mahieu & Schotman, 1998). Asset returns are leptokurtic, and reflect volatility clustering (Chernov, Gallant, Ghysels, & Tauchen, 1999). As economic environments change, so do the data generating processes of related financial variables. Empirical results in the studies of Eraker, Johannes, and Polson (2003) and Bakshi, Cao, and Chen (1997) among others show that it is necessary to include jumps in the stochastic volatility in order to account for sudden changes in volatility. Bates (1996) and Scott (1997) combine stochastic volatility models with jumps in returns. Duffie, Pan, and Singleton (2000) and Raggi (2006) propose that SVJ models can well describe foreign exchange market volatility.

The inputs of SVR will include moneyness (spot rate/strike price), forward rate, domestic risk-free simple interest rate, time to maturity, and volatility of spot rate. The new model reduces forecasting errors of the parametric methods and performs better than ANN model.

The remaining part of this paper is as follows. In next section, the basic ideas and arithmetic of SVJ model and SVR are discussed. In Section 3, the new European currency options pricing model is proposed. In Section 4, the empirical study is performed, i.e., the new model is applied to China currency options market. And relative performance of new model is then analyzed by comparing its results with that of artificial network on the same data sets. Finally, some important conclusions and further study are stated in Section 5.

2. Methodology

2.1. Garman–Kohlhagen formula

Regarding price foreign exchange options, Garman and Kohlhagen (1983) assumed that spot rate follows the lognormal process. The framework is basically Black–Scholes model. In Garman and Kohlhagen formula for European call option, foreign exchange rate is assumed to follow the standard lognormal diffusion:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \tag{1}$$

where W_t is standard Brownian motion. A call currency option has value:

$$C(S,K,r,rf,T,\sigma) = Se^{-rf*t}N(d1) - Ke^{r*T}N(d2), \tag{2}$$

where

$$d1 = \frac{\ln(S/K) + (r - rf + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \tag{3}$$

$$d2 = d1 - \sigma\sqrt{T}. \tag{4}$$

S is the spot rate; K is the strike price; σ is the volatility of the spot rate; r is domestic risk-free simple interest rate; rf is foreign risk-free simple interest rate; T is the time to maturity (calculated according to the appropriate day count convention); $N(*)$ is the cumulative normal distribution function.

2.2. Stochastic volatility model with jumps (SVJ)

SV model is introduced by Taylor (1986) to describe financial time series. It offers a powerful alternative to the Garch-type models in explaining the well-documented time-varying volatility exhibited in many financial time series (Kim, Shephard, & Chib, 1998; Yu, 2002). Lognormal SV(LNSV) model is widely used. It assumes that volatility follows Ornstein–Uhlenbeck process, which is applied in derivatives pricing and interest rates. Many other SV models are studied (Barndorff-Nielsen & Shephard, 2001; Heston, 1993; Jones, 2003). In order to describe rare events like crashes in the market, the introduction of jumps in modeling financial returns seems appropriate (Raggi, 2006). Duffie et al. (2000) introduced the affine jump diffusion family. We consider models with jumps specification on the volatility equations.

A stochastic volatility model that belongs to the logarithmic class is:

$$y_t = \varepsilon_t \exp(h_t/2), \quad \varepsilon_t \sim IID(0,1), \quad t = 1,\ldots,T, \tag{5}$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma_\eta \eta_t, \quad \eta_t \sim NID(0,1), \tag{6}$$

where $|\varphi| < 1$, y_t is the asset return on day t from which the mean and autocorrelations are removed. In the following, $\exp(h_t/2)$ is called volatility, h_t represents the log of squared volatility, ε_t, η_t are noise processes, which are assumed to be independent of one another. Eq. (5) is an observed return equation, and Eq. (6) is a latent volatility equation.

Similar to the previous SV model, the Euler discretization of continuous time jump process leads to a specification of the form

$$y_t = \varepsilon_t \exp(h_t/2) + J_t N_t, \tag{7}$$
$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma_\eta \eta_t, \tag{8}$$
$$N_t \sim Ber(\lambda),$$
$$J_t \sim N(\mu, \sigma),$$

where N_t is the indicator of jump and J_t the jump size. $\lambda \sim Beta(a,b)$, $\mu \sim N(c,d)$ and $\sigma^2 \sim IG(v/2, v\sigma^2/2)$.

In this paper, the SVJ model we consider is the affine jump diffusion family introduced in Duffie et al. (2000) and Raggi (2006). The SVJ model is specified as following two equations:

$$dy_t = \left(\mu - \frac{1}{2}e^{h_T}\right)dt + e^{(1/2)h_t}dW_{1,t} + J_t^Y dN_t, \tag{9}$$

$$dh_t = \left[\kappa(\theta e^{-h_t} - 1) - \frac{1}{2}\sigma_v^2 e^{-h_t}\right]dt + \sigma_v e^{(1/2)h_t}dW_{2,t} + \log\left(1 + J_t^Y e^{-h_t}\right)dN_t, \tag{10}$$

where $(W_{1,t}, W_{2,t})$ are Brownian motions. N_t is a standard counting process. J_t^Y is the jump's size, and

$$J_t^Y dN_t = \sum_{k=1}^{dN_t} J_t^Y, . \, J^Y \Big/ J^N \sim N\left(\mu_j + \rho J^N, \sigma_j\right).$$

Unfortunately, it is a difficult task to estimate classical parameter for SV model due to intractable form of the likelihood function. A variety of estimation methods have been proposed for the SV model to overcome the difficulties, including general method of moments (GMM) (Melino & Turnbull, 1990), quasi-maximum likelihood (QML) (Harvey et al., 1994; Ruiz, 1994), efficient method of moments (EMM) (Gallan, Hsieh, & Tauchen, 1997), and Markov chain Monte Carlo (MCMC) procedures for the SV model (Kim et al., 1998). Among these methods MCMC is ranked as one of the most useful estimation tools.