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General equilibrium pricing of options with habit formation and event risks ☆

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ABSTRACT

This paper proposes a general equilibrium model that explains the pricing of the S&P 500 index options. The central ingredients are a peso component in the consumption growth rate and the time-varying risk aversion induced by habit formation which amplifies consumption shocks. The amplifying effect generates the excess volatility and a large jump-risk premium which combine to produce a pronounced volatility smirk for index options. The time-varying volatility and jump-risk premiums explain the observed state-dependent smirk patterns. Besides volatility smirks, the model has a variety of other implications which are broadly consistent with the aggregate stock and option market data.

1. Introduction

It is well-known that the Black and Scholes (1973, B/S) model cannot explain the observed index option data after the 1987 market crash. The biggest puzzle is the so-called volatility smirk. Options, including ATM options, are typically priced at a premium; and the premium is higher for OTM (out-of-the-money) put options than for ATM (at-the-money) options, generating a smirk pattern in the cross-sectional plot of the implied Black-Scholes volatility (B/S-vol) against the options' moneyness.[1] In addition, the smirk patterns tend to vary over changing economic conditions. None of the above empirical observations can be explained by the traditional Black-Scholes model in which the option-implied volatilities are equal to the return volatility of the underlying index which is assumed to be a constant.

This paper proposes a consumption-based explanation of both the average and the state dependences of the smirk patterns. I use a representative agent model with

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[1] Moneyness is defined as the ratio of the strike price of an option contract to the spot price of the underlying asset on which the option is written. Therefore, an ATM option would have the moneyness of one. By convention, OTM and ITM (in-the-money) puts refer to put options with moneyness less than and greater than one, respectively.

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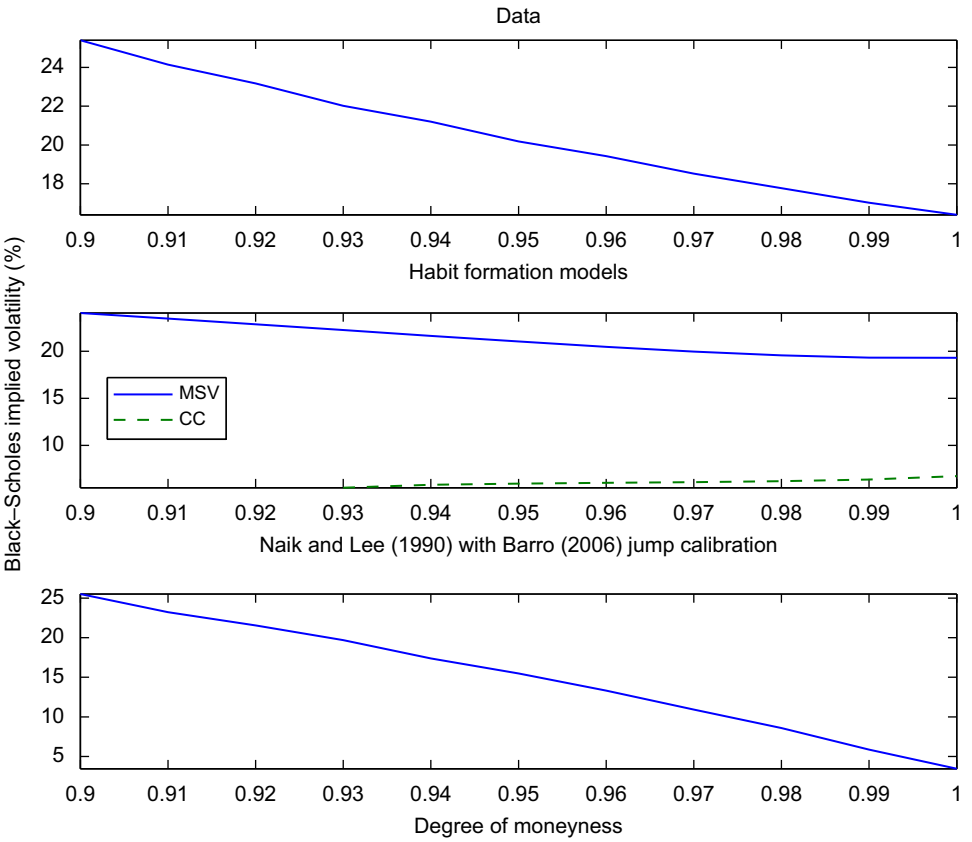

Fig. 1. Option pricing implied from the data and the previous models. This figure plots the implied volatility smirks for options with 30 days to expiration. The top panel plots the observed volatility smirk for the S&P 500 index option data averaged over the period from April 4, 1988 to September 30, 2008. The middle panel plots the volatility smirks implied from Campbell and Cochrane (1999, CC) and Menzly, Santos, and Veronesi (2004, MSV) under their original calibrations. The bottom panel plots the volatility smirk implied from Naik and Lee (1990) with Barro's (2006) choice of jump parameters.

non-time-separable preferences and time-varying risk aversion induced by habit formation. Aggregate consumption is exogenous and its instantaneous growth rate follows an i.i.d. lognormal process subject to a small-probability jump. The jump component models the rare economic disasters which strike at a constant intensity. Closed-form valuation is derived for the aggregate stock as the claim to the aggregate consumptions, and index options are proxied by options written on the aggregate stock. Within the representative agent framework, the model nests as special cases in both habit formation models (e.g., Campbell and Cochrane, 1999, CC; the aggregate model of Menzly, Santos, and Veronesi, 2004, MSV) and "peso problem"[2] models (e.g., Naik and Lee, 1990; Barro, 2006).

The main mechanism of the model is as follows. At the presence of habit formation, risk aversion reacts negatively to changes in the aggregate consumption creating an extra channel by which consumption innovations induce excess innovations in the stock market beyond and above those due to cash-flow innovations. Excess stock return innovations take two forms: excess diffusive volatility and the excess jumps. First, excess volatility generates high ATM prices which determine the levels of the observed volatility smirk. Second, positive risk-aversion jumps induced by the negative consumption jumps raise the marginal utility which makes stock market crashes particularly unpleasant. This effect, combined with the amplifying return jumps, generates a large jump-risk premium. Third, options with varying moneyness are sensitive to the potential jumps in a variety of ways. In particular, deep OTM puts are much more sensitive to market crashes than ATMs, and hence, bear higher levels of jump-risk premiums. Taken together, excess volatility and a large compensation for jump risks combine to generate the pronounced smirk pattern observed in the data which is plotted in the top panel of Fig. 1, where I use the data of S&P 500 index options with 30 days to expiration for the period from April 1988 to September 2008.

The model also predicts that the smirk premium, measured as the price (quoted in B/S-vol) difference between 10% OTM puts and ATMs, is decreasing in the underlying volatility. Except for very bad states which are unlikely to occur, both volatility and jumps in my model

[2] The term of "peso problem" is attributed to Milton Friedman's comments about the effects of the infrequent but disastrous events on the Mexican peso market in the early 1970s.