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Pricing decisions with retail competition in a fuzzy closed-loop supply chain

Jie Wei [a], Jing Zhao [b,*]

[a] General Courses Department, Academy of Military Transportation, Tianjin 300161, China
[b] School of Science, Tianjin Polytechnic University, Tianjin 300160, China

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ABSTRACT

The optimal pricing decision problem of a fuzzy closed-loop supply chain with retail competition is considered in this paper. The fuzziness is associated with the customer demands, the remanufacturing cost and the collecting cost. By using game theory and fuzzy theory, the optimal decision on wholesale price, retail prices and remanufacturing rate are explored respectively under the centralized and the decentralized decision scenarios, and the expressions for them are also established. Some insights into the economic behavior of firms are given, which can serve as the basis for further study in the future.

1. Introduction

Because of government legislations, customers' behaviors on the environment and organizations' attitudes from the economic point, the reverse logistics systems as an environmentally friendly policies have been adopted in many countries, and the interest in reverse logistics has attracted the attention of not only companies and professionals but also academia, which has been tackling this issue in recent years (Prahinski & Kocabasoglu, 2006). Many researches have been done about the reverse logistics and closed-loop supply chain (Guide & Harrison, 2003; Teunter, 2004; Mahadevan, Pyke, & Fleischmann, 2003; Mitra, 2009).

In fact, in order to make effective closed-loop supply chain management, the uncertainties that happen in the real world cannot be ignored. Those uncertainties are usually associated with the product supply, the customer demand, used products' collecting and so on. The quantitative demand forecasts based on manager's judgements, intuitions and experience seem to be more appropriate. Fuzzy theory, originally introduced by Zadeh (1965), provides a reasonable way to deal with the possibility and linguistic expressions (i.e. decision maker's judgements, for example, remanufacturing cost may be expressed as "low cost" or "high cost" to make rough estimates, market base can be expressed as "large market base" or "small market base" to make rough estimates).

In recent supply chain studies, some researchers have already adopted fuzzy theory to depict uncertainties in supply chain models. Petrovic, Roy, and Petrovic (1999) and Petrovic (2001) formulated an inventory model with fuzzy customer's demand and fuzzy supply of raw materials to investigate order quantities for each inventory in the supply chain. Giannaoccaro, Pontrandolfo, and Scozzi (2003) considered the echelon stock policy with fuzzy market demand and fuzzy inventory costs. Gumus and Guneri (2009) considered the problem of a multi-echelon inventory management framework of stochastic and fuzzy supply chains. An inventory management framework and deterministic/stochastic-neuro-fuzzy cost models are structured in this paper. Xie, Petrovic, and Burnham (2006) presented a new hierarchical, two-level approach to inventory management and control in supply chains under fuzzy customer demand. Zhang, Zhao, and Tang (2009) presented a random fuzzy economic manufacturing quantity model in a deteriorating process. The setup cost and the average holding cost are characterized as fuzzy variables and the elapsed time until shift is a random fuzzy variable. Zhao, Tang, and Wei (2011) studied the pricing problem of two substitutable products in a supply chain with one manufacturer and two competitive retailers. The consumer demands and manufacturing costs are of uncertainty, which are described by fuzziness. Wang (2009) studied the inventory control of a DRP's (Distribution Requirement Planning) supply chain management with fuzzy theory. A continuous review model is considered and a new method on the model with triangular fuzzy numbers is also presented in this paper.

This paper differs from the previous ones in that we consider a pricing decisions model for a fuzzy closed-loop supply chain with retail competition in the marketplace. The fuzziness is associated with the consumer demand, the remanufacturing cost, and the collecting cost. The purpose of this paper is to explore how each firm make the optimal decision facing fuzzy environment. By using game theory and fuzzy theory, the closed-form expressions for the optimal decisions are established, through which we can see that how the manufacturer and the two competitive retailers make their own decisions about wholesale price, collecting rate and retail prices, respectively, in the fuzzy expected value model. Some managerial insights are derived by using the models established in this paper.

* Corresponding author.
E-mail address: zhaojing0006@163.com (J. Zhao).

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The rest of the paper is organized as follows. Section 2 gives the preliminaries for this paper. Section 3 gives the model description and notations, and Section 4 details our key analytical results. Fuzzy simulation and numerical studies are given in Section 5. Concluding remarks are presented in Section 6.

2. Preliminaries

A possibility space is defined as a triplet $(\Theta, \mathcal{P}(\Theta), \text{Pos})$, where Θ is a nonempty set, $\mathcal{P}(\Theta)$ the power set of Θ, and Pos a possibility measure. Each element in $\mathcal{P}(\Theta)$ is called an fuzzy event. For each event A, Pos$\{A\}$ indicates the possibility that A will occur. Nahmias (1978) gave the following three axioms.

Axiom 1.

$\text{Pos}\{\Theta\} = 1.$

Axiom 2. Pos$\{\phi\}$ = 0, where ϕ denotes the empty set.

Axiom 3. $\text{Pos}\{\bigcup_{i=1}^m A_i\} = \sup_{1\leqslant i\leqslant m}\text{Pos}\{A_i\}$ for any collection A_i in $\mathcal{P}(\Theta)$.

Besides the axioms mentioned above, there is another axiom given by Liu (2002) to define the product possibility space.

Axiom 4. Let Θ_i be nonempty sets, on which Pos$_i$ is possibility measure satisfying the first three axioms, $i = 1,2,\ldots,n$, and $\Theta = \prod_{i=1}^n \Theta_i$. Then

$$\text{Pos}\{A\} = \sup_{(\theta_1,\theta_2,\ldots,\theta_n)\in A} \text{Pos}_1\{\theta_1\} \wedge \text{Pos}_2\{\theta_2\} \wedge \cdots \wedge \text{Pos}_n\{\theta_n\},$$

for each $A \in \mathcal{P}(\Theta)$. In that case we write $\text{Pos} = \wedge_{i=1}^n \text{Pos}_i$.

Lemma 1 (Liu, 2002). *Suppose that $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)$ is a possibility space, $i = 1,2,\ldots,n$. By Axiom 4, $(\prod_{i=1}^n \Theta_i, \mathcal{P}(\prod_{i=1}^n \Theta_i), \wedge_{i=1}^n \text{Pos}_i)$ is also a possibility space, which is called the product possibility space.*

Definition 1 (Nahmias, 1978). A fuzzy variable is defined as a function from the possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ to the set of real numbers and its membership function is derived from the possibility by

$$\mu_\xi(x) = \text{Pos}\{\theta \in \Theta | \xi(\theta) = x\}, \quad x \in R.$$

Definition 2 (Liu, 2002). A fuzzy variable ξ is said to be nonnegative (or positive) if Pos$\{\xi < 0\}$ = 0 (or Pos$\{\xi \leqslant 0\}$ = 0).

Definition 3 (Liu, 2002). Let $f : R^n \to R$ be a function, and ξ_i be fuzzy variable defined on the possibility space $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)$, $i = 1,2,\ldots,n$, respectively. Then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is a fuzzy variable defined on the product possibility space $(\prod_{i=1}^n \Theta_i, \mathcal{P}(\prod_{i=1}^n \Theta_i), \wedge_{i=1}^n \text{Pos}_i)$ as $\xi(\theta_1, \theta_2, \ldots, \theta_n) = f(\xi_1(\theta_1), \xi_2(\theta_2), \ldots, \xi_n(\theta_n))$ for any $(\theta_1, \theta_2, \ldots, \theta_n) \in \prod_{i=1}^n \Theta_i$.

The independence of fuzzy variables was discussed by several researchers, such as Zadeh (1978), Nahmias (1978) and Liu (2002).

Definition 4. The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_n$ are independent if for any sets $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n$ of R,

$$\text{Pos}\{\xi_i \in \mathcal{B}_i, i = 1,2,\ldots,n\} = \min_{1\leqslant i\leqslant n} \text{Pos}\{\xi_i \in \mathcal{B}_i\}.$$

Lemma 2 (Liu, 2004). *Let ξ_i be independent fuzzy variable, and $f_i : R \to R$ function, $i = 1,2,\ldots,m$. Then $f_1(\xi_1), f_2(\xi_2), \ldots, f_m(\xi_m)$ are independent fuzzy variables.*

Definition 5 (Liu, 2002). Let ξ be a fuzzy variable on the possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$, and $\alpha \in (0,1]$. Then

$$\xi_\alpha^L = \inf\{r | \text{Pos}\{\xi \leqslant r\} \geqslant \alpha\} \quad \text{and} \quad \xi_\alpha^U = \sup\{r | \text{Pos}\{\xi \geqslant r\} \geqslant \alpha\}$$

are called the α-pessimistic value and the α-optimistic value of ξ, respectively.

Example 1. The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has its α-pessimistic value and α-optimistic value

$$\xi_\alpha^L = a_2\alpha + a_1(1-\alpha) \quad \text{and} \quad \xi_\alpha^U = a_2\alpha + a_3(1-\alpha).$$

Lemma 3 (Wang, Tang, & Zhao, 2007). *Let ξ_i be independent fuzzy variables defined on the possibility spaces $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)$ with continuous membership function, $i = 1,2,\ldots,n$, and $f : X \subset \mathcal{R}^n \to \mathcal{R}$ a measurable function. If $f(x_1, x_2, \ldots, x_n)$ is monotonic with respect to x_i, respectively, then*

(a) $f_\alpha^U(\xi) = f(\xi_{1\alpha}^V, \xi_{2\alpha}^V, \ldots, \xi_{n\alpha}^V)$, *where* $\xi_{i\alpha}^V = \xi_{i\alpha}^U$, *if* $f(x_1, x_2, \ldots, x_n)$ *is nondecreasing with respect to* x_i; $\xi_{i\alpha}^V = \xi_{i\alpha}^L$, *otherwise,*
(b) $f_\alpha^L(\xi) = f(\xi_{1\alpha}^{\overline{V}}, \xi_{2\alpha}^{\overline{V}}, \ldots, \xi_{n\alpha}^{\overline{V}})$, *where* $\xi_{i\alpha}^{\overline{V}} = \xi_{i\alpha}^L$, *if* $f(x_1, x_2, \ldots, x_n)$ *is nondecreasing with respect to* x_i; $\xi_{i\alpha}^{\overline{V}} = \xi_{i\alpha}^U$, *otherwise,where* $f_\alpha^U(\xi)$ *and* $f_\alpha^L(\xi)$ *denote the* α*-optimistic value and the* α*-pessimistic value of the fuzzy variable* $f(\xi)$*, respectively.*

Definition 6 (Liu & Liu, 2002). Let $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ be a possibility space and A a set in $\mathcal{P}(\Theta)$. The credibility measure of A is defined as

$$\text{Cr}\{A\} = \frac{1}{2}(1 + \text{Pos}\{A\} - \text{Pos}\{A^c\}),$$

where A^c denotes the complement of A.

Definition 7 (Liu & Liu, 2002). Let ξ be a fuzzy variable. The expected value of ξ is defined as

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geqslant x\}\mathrm{d}x - \int_{-\infty}^0 \text{Cr}\{\xi \leqslant x\}\mathrm{d}x$$

provided that at least one of the two integrals is finite.

Example 2. The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has an expected value

$$E[\xi] = \frac{a_1 + 2a_2 + a_3}{4}.$$

Definition 8 (Liu & Liu, 2002). Let f be a function on $R \to R$ and ξ be a fuzzy variable. Then the expected value $E[f(\xi)]$ is defined as

$$E[f(\xi)] = \int_0^{+\infty} \text{Cr}\{f(\xi) \geqslant x\}\mathrm{d}x - \int_{-\infty}^0 \text{Cr}\{f(\xi) \leqslant x\}\mathrm{d}x$$

provided that at least one of the two integrals is finite.

Lemma 4 (Liu & Liu, 2003). *Let ξ be a fuzzy variable with finite expected value. Then*

$$E[\xi] = \frac{1}{2}\int_0^1 (\xi_\alpha^L + \xi_\alpha^U)\mathrm{d}\alpha.$$