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Nonlinear gray-box identification using local models applied to industrial robots☆

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ABSTRACT

In this paper, we study the problem of estimating unknown parameters in nonlinear gray-box models that may be multivariable, nonlinear, unstable, and resonant at the same time. A straightforward use of time-domain predication-error methods for this type of problem easily ends up in a large and numerically stiff optimization problem. We therefore propose an identification procedure that uses intermediate local models that allow for data compression and a less complex optimization problem. The procedure is based on the estimation of the nonparametric frequency response function (FRF) in a number of operating points. The nonlinear gray-box model is linearized in the same operating points, resulting in parametric FRFs. The optimal parameters are finally obtained by minimizing the discrepancy between the nonparametric and parametric FRFs. The procedure is illustrated by estimating elasticity parameters in a six-axis industrial robot. Different parameter estimators are compared and experimental results show the usefulness of the proposed identification procedure. The weighted logarithmic least squares estimator achieves the best result and the identified model gives a good global description of the dynamics in the frequency range of interest for robot control.

1. Introduction

When building a mathematical model of a physical object, the user generally has two sources of information; prior knowledge and experimental data. This gives the two modeling extremes, *white-box models* that are the result of extensive physical modeling from first principles, and *black-box models*, where the model is just a vehicle to describe the experimental data without any physical interpretations of its parameters. In between comes gray-box models that are parameterizations based on various degrees of physical insights. Compared with identification methods using black-box models, gray-box models have some particular benefits. By including prior information, the model set (number of parameters) can be reduced while still providing a good approximation of the true system. This gives a reduced mean-square error (bias and variance) (Ljung, 2008). Since the gray-box model in contrast to the black-box model has a physical interpretation, the model is also useful in many ways, such as design optimization and virtual prototyping, which are important areas in industry. Some standard references for identification of gray-box models are Bohlin (2006), Ljung (1999), and Schittkowski (2002). There are also software packages available for identifying such models, e.g., Bohlin (2006), Kristensen, Madsen, and Jørgensen (2004), Ljung (2007), and Schittkowski (2002).

In this paper, we will mainly consider identification of nonlinear gray-box models with the lightest shade of gray, where a white-box model contains some unknown or uncertain parameters that need to be estimated from data. To be even more specific, we will allow our systems to be *multivariable*, *nonlinear*, *unstable*, and *resonant* at the same time. Usually, in the literature, at least one of the first three properties is left out to reduce the problem complexity. Identification of such complex systems is therefore challenging, both in finding suitable model structures and efficient identification methods. A straightforward use of time-domain predication-error methods (Ljung, 1999) for this type of system easily ends up in a large optimization problem, where each iteration in the optimization routine involves a number of simulations of a large and numerically stiff ODE for many samples. We therefore propose an identification procedure that uses intermediate local models that allow for data compression and a less complex optimization problem. The procedure is based on the estimation of the nonparametric frequency response function (FRF) in a number of operating points. The nonlinear gray-box model is linearized in the same operating points, resulting in parametric FRFs. The optimal parameters are finally obtained by minimizing a parametric criterion,

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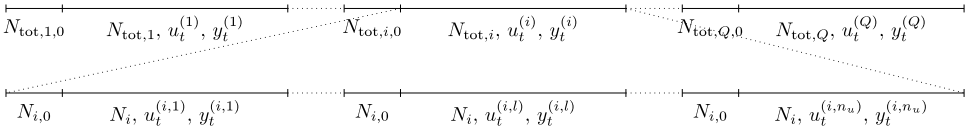

Fig. 1. Illustration how the N_{tot} samples are distributed into Q operating points, where $N_{\text{tot},i,0}$ samples are "wasted" to get to operating point i where the $N_{\text{tot},i}$ samples are further distributed into n_u parts to estimate a nonparametric FRF by using the n_u steady-state responses, $u_t^{(i,1)}, y_t^{(i,1)}, \ldots, u_t^{(i,n_u)}, y_t^{(i,n_u)}$, each of length N_i.

measuring the discrepancy between the nonparametric and parametric FRFs.

The main motivation for our study is identification of accurate dynamic models for industrial robots, which incorporate all the four mentioned properties. We will come back to this application in Sections 5 and 6 when identification of elasticity parameters (spring–damper pairs) is used as an example of the proposed identification procedure.

Various aspects of the identification procedure will now be explained in the coming sections, starting with a more detailed outline of the procedure in Section 2. The nonparametric FRF estimation is described in Section 3, and some variants of the parametric criterion are presented in Section 4. This is followed by an experimental evaluation in Section 5 and a small comparison with time domain identification in Section 6. Finally, Section 7 concludes the paper.

2. Gray-box identification using local models

In this paper we consider the problem of identifying parameters θ in the following nonlinear gray-box model

$$x_{t+1} = f(x_t, u_t, \theta) + g(x_t, u_t, \theta)w_t, \tag{1a}$$

$$y_t = h(x_t, u_t, \theta) + e_t, \tag{1b}$$

with state vector $x_t \in \mathbb{R}^{n_x}$, input vector $u_t \in \mathbb{R}^{n_u}$, output vector $y_t \in \mathbb{R}^{n_y}$, and $\theta \in \Theta \subset \mathbb{R}^{n_\theta}$ a vector of unknown parameters that specifies the mappings $f(\cdot), g(\cdot), h(\cdot)$ that may all be nonlinear. Furthermore, w_t and e_t are process- and measurement noise vectors, that are assumed to be mutually independent zero-mean white processes with covariance matrices $\mathrm{E}(w_t w_t^T) = Q_{\text{w}}$ and $\mathrm{E}(e_t e_t^T) = Q_e$.

One solution to the problem of identifying θ in (1) is to apply a nonlinear prediction error method (Ljung, 1999, pp. 146–147). The idea then is to find the parameters that will minimize the prediction errors

$$\varepsilon_{t,\theta} = y_t - \hat{y}_{t,\theta}, \quad t = 1, 2, \ldots, N_{\text{tot}},$$

where $\hat{y}_{t,\theta}$ is the model's prediction of y_t given previous measurements $\{u_1, y_1, \ldots, u_{t-1}, y_{t-1}, u_t\}$. For the minimization, one can choose different norms, but a common choice is a quadratic criterion

$$V(\theta) = \sum_{t=1}^{N_{\text{tot}}} \varepsilon_{t,\theta}^T \Lambda_t^{-1} \varepsilon_{t,\theta}, \tag{2}$$

with weighting matrix Λ_t and N_{tot} equal to the total number of measurement samples. For a general nonlinear system as in (1), it is often very hard to determine a predictor on formal probabilistic grounds. In most cases there is no explicit form available for the optimal solution. This implies that a predictor must be constructed either by *ad hoc* approaches, or by some approximation of the optimal solution, e.g., by using sequential Monte Carlo techniques (Doucet et al., 2001) or the Extended Kalman Filter (Anderson & Moore, 1979). We will further discuss such solutions in Section 2.4.

In this paper, we propose another solution to handle the nonlinearities in the system, where θ is identified by using intermediate local models. By performing experiments that sequentially excite the local system behavior in a number of operating points $(u_0^{(i)}, y_0^{(i)})$, $i = 1, \ldots, Q$, the criterion (2) can be approximated by

$$V(\theta) = \sum_{i=1}^{Q} \sum_{t=1}^{N_{\text{tot},i}} [\varepsilon_{t,\theta}^{(i)}]^T [\Lambda_t^{(i)}]^{-1} \varepsilon_{t,\theta}^{(i)}, \tag{3}$$

using the notation $\varepsilon_{t,\theta}^{(i)} = y_t^{(i)} - \hat{y}_{t,\theta}^{(i)}$ where $y_t^{(i)} = y_{t+t_i} - y_0^{(i)}$ denotes the measured output around operating point i with output vector $y_0^{(i)}$. The N_{tot} samples in (2) are therefore distributed according to Fig. 1 (top part), where $N_{\text{tot},i,0}$ samples are needed to move the system to operating point i, in which $N_{\text{tot},i}$ samples are collected, such that $t_i = N_{\text{tot},i,0} + \sum_{r=1}^{i-1}(N_{\text{tot},r} + N_{\text{tot},r,0})$ and $N_{\text{tot}} = \sum_{i=1}^{Q}(N_{\text{tot},i} + N_{\text{tot},i,0})$.

2.1. Linearized gray-box model

The predictor $\hat{y}_{t,\theta}^{(i)}$ for the local model can be simplified compared to $\hat{y}_{t,\theta}$ since only the local (linear) behavior needs to be captured. Assume now that the behavior of (1) around an operating point $(u_0^{(i)}, y_0^{(i)})$ can be described by the linearized model

$$x_{t+1}^{(i)} = A_\theta^{(i)} x_t^{(i)} + B_\theta^{(i)} u_t^{(i)} + B_{\text{w},\theta}^{(i)} w_t^{(i)}, \tag{4a}$$

$$y_t^{(i)} = C_\theta^{(i)} x_t^{(i)} + D_\theta^{(i)} u_t^{(i)} + e_t^{(i)}, \tag{4b}$$

where $x_t^{(i)} = x_{t+t_i} - x_0^{(i)}$, $u_t^{(i)} = u_{t+t_i} - u_0^{(i)}$, $y_t^{(i)} = y_{t+t_i} - y_0^{(i)}$, and $x_0^{(i)}$ is obtained as, e.g., the solution to $x_0^{(i)} = f(x_0^{(i)}, u_0^{(i)}, \theta)$, $y_0^{(i)} = h(x_0^{(i)}, u_0^{(i)}, \theta)$. Note that $x_0^{(i)}$ therefore, in general, will be θ-dependent. The matrices $A_\theta^{(i)}, B_\theta^{(i)}, C_\theta^{(i)}, D_\theta^{(i)}$ are the partial derivatives of $f(\cdot)$ and $h(\cdot)$ w.r.t. x_t and u_t and evaluated in $(x_0^{(i)}, u_0^{(i)})$, and $B_{\text{w},\theta}^{(i)} = g(x_0^{(i)}, u_0^{(i)}, \theta)$.

By assuming w_t and e_t to be Gaussian and neglecting transients, the optimal one-step ahead predictor $\hat{y}_{t,\theta}^{(i)}$ for (4) is given by the steady-state Kalman filter

$$\hat{x}_{t+1}^{(i)} = A_\theta^{(i)} \hat{x}_t^{(i)} + B_\theta^{(i)} u_t^{(i)} + K_\theta^{(i)} (y_t^{(i)} - \hat{y}_{t,\theta}^{(i)}), \tag{5a}$$

$$\hat{y}_{t,\theta}^{(i)} = C_\theta^{(i)} \hat{x}_t^{(i)} + D_\theta^{(i)} u_t^{(i)}. \tag{5b}$$

Here, $K_\theta^{(i)} = A_\theta^{(i)} P_\theta^{(i)} [C_\theta^{(i)}]^T [C_\theta^{(i)} P_\theta^{(i)} [C_\theta^{(i)}]^T + Q_e]^{-1}$ where $P_\theta^{(i)}$ is the positive semi-definite solution of the stationary Riccati equation (omitting indices)

$$P = APA^T - APC^T(CPC^T + Q_e)^{-1}CPA^T + B_{\text{w}} Q_{\text{w}} B_{\text{w}}^T.$$

By using (5), the prediction error $\varepsilon_{t,\theta}^{(i)}$ can be written as

$$\varepsilon_{t,\theta}^{(i)} = [H_\theta^{(i)}(q)]^{-1}[y_t^{(i)} - G_\theta^{(i)}(q) u_t^{(i)}], \tag{6}$$

where q is the difference operator, $u_{t+1} = q u_t$, and

$$G_\theta^{(i)}(q) = C_\theta^{(i)}[qI - A_\theta^{(i)}]^{-1} B_\theta^{(i)} + D_\theta^{(i)}, \tag{7a}$$

$$H_\theta^{(i)}(q) = C_\theta^{(i)}[qI - A_\theta^{(i)}]^{-1} K_\theta^{(i)} + I. \tag{7b}$$

In case the measurement data around operating point i can be described by (4) with $\theta = \theta_0$, the prediction error $\varepsilon_{t,\theta_0}^{(i)}$ is the innovation with covariance matrix $\Lambda_0^{(i)} = C_{\theta_0}^{(i)} P_{\theta_0}^{(i)} [C_{\theta_0}^{(i)}]^T + Q_e$. The weights $\Lambda_t^{(i)}$ in (3) should therefore be close to $\Lambda_0^{(i)}$.