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# Theory and application of conflict resolution with hybrid preference in colored graphs ☆

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ABSTRACT

An algebraic approach is proposed to calculate stabilities in a colored graph with hybrid preference. The algebraic approach establishes a hybrid framework for stability analysis by combining strength of preference and unknown preference. The hybrid system is more general than existing models, which consider preference strength and preference uncertainty separately. Within the hybrid preference structure, matrix representations of four basic stabilities in a colored graph are extended to include mild, strong, and uncertain preference and algorithms are developed to calculate efficiently the inputs essential to the stability definitions. A specific case study, including multiple decision makers and hybrid preference, is used to illustrate how the proposed method can be applied in practice.


## 1. Introduction

An innovative system based on matrices is developed to represent hybrid preference and calculate corresponding stabilities in a graph model. Hybrid preference extends existing preference structures to combine unknown preference and strength of preference into the graph model for conflict resolution (GMCR) [1]. The three existing preference structures, simple preference [1], preference with strength [2,3], and unknown preference [4], are integrated into a new hybrid preference framework for enhancing the applicability of graph models. This structure uses a quadruple relation to express equal preference, mild preference, or strong preference of one state or scenario over another, plus unknown preference.

The logical representation of the four basic stabilities, Nash stability [5,6], general metarationality (GMR) [7], symmetric metarationality (SMR) [7], and sequential stability (SEQ) [8], under hybrid preference was developed by Xu et al. [9]. As was observed in the book by Fang et al. [1], procedures to identify stable states based on logical definitions are difficult to code because of the nature of the logical representations. Although hybrid preference was included in the graph model for conflict resolution with multiple decision makers (DMs) [9], the corresponding algorithm to implement stability analysis has not been developed because of its logical complexity. To overcome this limitation, the matrix representations of the four basic stabilities in multiple-decision-maker graph models were developed for simple preference [10], unknown preference [11], and for strength of preference [12].

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A graph model is a representation of a conflict including the DMs, those feasible states, each DM's preference on these states, and the movements controlled by each DM. These movements can be drawn as a directed graph, with states as vertices, for each DM, or as a single integrated graph. An important restriction of the graph model system is that no DM can move twice in succession along any path. Therefore, a graph model can be treated as an edge-colored multi-digraph in which each arc represents a legal unilateral move (UM) and distinct colors refer to different DMs, who may control the move [13]. Tracing the evolution of a conflict can thus be converted to searching all colored paths from an initial state to a particular outcome in an edge-colored digraph. A graph model with hybrid preference (strength of preference and unknown preference) can equally well be represented as a colored multi-digraph.

In this paper, matrix representation of the four stabilities under hybrid preference for multiple DMs is formulated explicitly in terms of matrices and several well-known results of Algebraic Graph Theory are used to analyze a graph model. The matrix representation effectively converts stability analysis from a logical procedure to an algebraic system. The algebraic approach can represent graph models with hybrid preference and support calculation of the four basic stabilities for such models. If one considers neither strength nor uncertainty of preference, stability analysis with hybrid preference will reduce to the corresponding solution concepts under simple preference [10]. When DMs' preferences do not include uncertainty, stabilities under hybrid preference reduce to the solution concepts under preference with strength only [12]; when a DM's preferences do not include strength, hybrid preference reduces to unknown preference [11]. Therefore, the hybrid preference structure offers analysts a more flexible mechanism for expressing a DM's preference. The development of the hybrid preference framework expands the realm of applicability of the graph model and provides new insights into strategic conflicts, so that more practical and complicated problems can be analyzed at greater depth. The algebraic approach facilitates the development of new stability concepts and algorithms to implement them. Hence, it will permit the design of the corresponding integrated decision support system to make decisions for various conflict events.

The rest of the paper is organized as follows. Section II presents the matrix representation of the hybrid preference for the colored-graph. Matrix representation of the four basic stabilities incorporating hybrid preference is presented in Section III. In Section IV, a conflict arising over proposed bulk water export from Lake Gisborne, located in Newfoundland, Canada, is modeled using the hybrid preference structure; then the proposed approach is demonstrated in practice. The paper concludes with some comments in Section V.

## 2. Matrix representation of combining strength of preference and unknown preference

A graph model is a structure $G\langle(S,A),N,P,c\rangle$, where $N$ is a non-empty set of DMs, $S$ and $A$ are the sets of states and arcs, respectively, and $(S,A)$ is a digraph with preference relations $P$ between any two states and a function $c : A \to N$ such that $c(a) \in N$ is the DM controlling the arc $a \in A$, provided that multiple edges of $(S,A)$ are controlled by different DMs, i.e., if $a \neq b$, but $a = (u,v)$ and $b = (u,v)$, then $c(a) \neq c(b)$. It means that a graph model may contain several arcs with the same initial and terminal states, but each arc is controlled by different DMs. Assuming that the state set $S$ and the DM set $N$ are numbered as $S = \{1,2,\ldots,m\}$ and $N = \{1,2,\ldots,n\}$, respectively. Similarly, by the proposed Rule of Priority [13], the oriented arcs of the arc set $A$ in the graph model are labeled according to the following priority order: the DM order; within each DM, according to the sequence of initial states; and within each DM and initial state, according to the sequence of terminal states. Hence, a graph model is converted into a colored graph with the labeled arcs when different DMs are colored different colors.

### 2.1. Framework of hybrid preference

In a graph model, preference plays an important role. To date, four types of preference structures–simple preference [1], preference possibly including uncertainty [4], preference having strength [2], and hybrid preference [9] – have been integrated into GMCR. The hybrid preference is defined using a quadruple relation $\{\sim_i, >_i, \gg_i, U_i\}$ in a graph model for DM $i$, in which preference relations, "$\sim_i$", "$>_i$", "$\gg_i$", and "$U_i$", denote DM $i$ indifferent, mildly preferred, strongly preferred, and uncertain, respectively. The hybrid structure is complete, i.e. if $s,q \in S$, then exactly one of the following relations holds: $s{\sim_i}q, s{>_i}q, q{>_i}s, s{\gg_i}q, q{\gg_i}s$, or $sU_iq$. For hybrid preference, DM $i$ can control six corresponding reachable lists from state $s$, which are $R_i^{++}(s), R_i^{+}(s), R_i^{U}(s), R_i^{=}(s)$, $R_i^{-}(s)$, and $R_i^{--}(s)$. The reachable lists of DM $i$ under hybrid preference are defined as follows:

(i) $R_i^{++}(s) = \{q \in S : (s,q) \in A_i \text{ and } q{\gg_i}s\}$ stands for DM $i$'s reachable list from state $s$ by a strong unilateral improvement (UI), where $A_i$ indicates that DM $i$ can make a unilateral move (UM) in one step from the initial state to the terminal state of the arc. This set contains all states $q$ which are strongly preferred by DM $i$ to state $s$ and can be reached in one step from $s$;
(ii) $R_i^{+}(s) = \{q \in S : (s,q) \in A_i \text{ and } q{>_i}s\}$ denotes DM $i$'s reachable list from state $s$ by a mild UI;
(iii) $R_i^{U}(s) = \{q \in S : (s,q) \in A_i \text{ and } qU_is\}$ denotes DM $i$'s reachable list from state $s$ by an uncertain move;
(iv) $R_i^{=}(s) = \{q \in S : (s,q) \in A_i \text{ and } q{\sim_i}s\}$ denotes DM $i$'s reachable list from state $s$ by an equally UM;
(v) $R_i^{-}(s) = \{q \in S : (s,q) \in A_i \text{ and } s{>_i}q\}$ denotes DM $i$'s reachable list from state $s$ by a mild unilateral disimprovement;
(vi) $R_i^{--}(s) = \{q \in S : (s,q) \in A_i \text{ and } s{\gg_i}q\}$ is DM $i$'s reachable list from state $s$ by a strong unilateral disimprovement.