Available at
www.ComputerScienceWeb.com
POWERED BY SCIENCE @ DIRECT®

Applied Soft Computing 2/3F (2003) 197–210

Applied Soft Computing

www.elsevier.com/locate/asoc

A bi-criteria optimization: minimizing the integral value and spread of the fuzzy makespan of job shop scheduling problems

Omar A. Ghrayeb*

Northern Illinois University, DeKalb, IL 60115, USA

Abstract

The processing times in reality are often uncertain and this uncertainty is critical for the scheduling procedures. This article presents bi-criteria genetic algorithm approach to solve fuzzy job shop scheduling problems (JSSPs), in which the integral value and the uncertainty of the fuzzy makespan (FM), which are conflicting objectives, are minimized. In this approach, imprecise processing times are modeled as triangular fuzzy numbers (TFNs), which results in a makespan that is a triangular fuzzy number. Therefore, it is practically important to pay attention to the uncertainty of the FM. Fuzzified benchmark problems; FT 6×6, La12, La13, and La14 were used to show the effectiveness of the proposed approach compared with optimizing fuzzy JSSPs with respect to either the FM or the uncertainty separately. A sensitivity analysis is also presented that shows the effect of the vagueness (uncertainty) of the processing times on the uncertainty of the FM.

Keywords: Scheduling; Genetic algorithms; Fuzzy numbers; Job shop

1. Introduction

Many researchers who deal with job shop scheduling problems (JSSPs) assume that time parameters, i.e. processing times, are fixed and deterministic. This assumption may be realistic if the operations under considerations are fully automated. However, whenever there is human interaction, this assumption may present difficulties in applying the schedule or even invalidating it. For example, the concept of processing time includes setup time and traveling between machines. Setup time and traveling time will not be exactly the same from day to day. Unfortunately, the uncertainty in these parameters has not received enough attention in [9–11].

Recently, researchers start to address the uncertainty of the data in the real world (i.e. processing times, due dates) and use fuzzy numbers to address this uncertainty. The first significant application that considers the uncertainty in time parameters is the one of Fortemps [5]. In this application the author used six-point fuzzy numbers to represent fuzzy durations. He used simulated annealing (SA) as an optimization technique and the optimization criterion was to minimize the fuzzy makespan (FM). To test the approach, he fuzzified the FT 6×6 problem [4] and other famous problems such as La11, La12, La13, and La14 [13]. The produced solutions are flexible, since they are able to cope with all possible durations within the specified range.

Chanas and Kasperski [2] considered fuzzy processing times and fuzzy due dates in the case of single machine scheduling problem. They show that Lawler's algorithm can be applied fuzzy scheduling problems on a single machine. Sakawa and Kubota [15] presented a two-objective genetic algorithm to minimize

* Tel.: +815-753-5660.
E-mail address: ghrayeb@ceet.niu.edu (O.A. Ghrayeb).

1568-4946/02/$ – see front matter © 2002 Elsevier Science B.V. All rights reserved.
PII: S1568-4946(02)00069-8

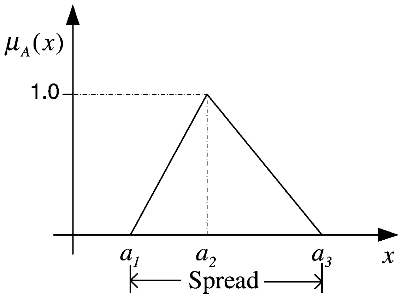

Fig. 1. Triangular fuzzy number A (a_1, a_2, a_3).

the maximum fuzzy completion time and maximize the average agreement index.

Ghrayeb [8] presented a genetic algorithm approach to optimizing fuzzy JSSPs, in which imprecise processing times are modeled as triangular fuzzy numbers (TFNs). This approach relies on using three-point fuzzy numbers to represent the imprecision in processing times. In that paper, the strength of his approach is that the produced schedule is flexible; it stays valid and can cope with all possible durations within the specified ranges. The objective (fitness) function considered was to minimizing the fuzzy makespan.

However, the choice what objective function to use depends on the application environment. If the application is within an environment where time is costly, we may prefer to minimize the fuzzy makespan. On the hand, if the application is within a just-in-time environment, we may prefer to minimize the uncertainty of the makespan measured by its spread. That is, it is practically important to pay attention to the uncertainty of the fuzzy makespan. This uncertainty can be measured by the spread of the triangular fuzzy number that represents the fuzzy makespan as shown in Fig. 1.

This article presents bi-criteria genetic algorithm approach to solve fuzzy JSSPs, in which the integral value and the uncertainty of the fuzzy makespan are minimized. In this approach, imprecise processing times are modeled as triangular fuzzy numbers, which results in a makespan that is modeled as a triangular fuzzy number.

2. Problem definition

The JSSP can be stated as follows: a set of n jobs $\{J_1, \ldots, J_n\}$ have to be processed on a set of m machines $\{M_1, \ldots, M_m\}$. Each job consists of a chain of operations and the operation order on machines is prespecified. The required machine and the processing time characterize each operation. In this problem, there are several constraints on both jobs and machines [1]. Given that processing times are uncertain (fuzzy), the goal for this problem is to determine the operation sequences on the machines that minimize the total integral value and the uncertainty of the fuzzy makespan, i.e. the time required to complete all jobs.

In JSSP, using fuzzy numbers to represent the uncertainty in processing times is very plausible. If a decision-maker in a plant estimates in terms of an interval rather than a single deterministic value, this induces directly the descriptions of the triangular fuzzy number A in Fig. 1. That is, the actual duration may be equal to a_2 (most plausible value), shorter (up to the optimistic value a_1) or longer (up to pessimistic value a_3) than a_2. However, the actual duration is more possible to be longer than it is shorter than a_2. Therefore, using a shifted triangular fuzzy number to model the uncertainty is more realistic.

Since processing times are modeled as triangular fuzzy numbers, two fuzzy operations are needed to calculate the completion times of different jobs on different machines. These two operations are the fuzzy sum and fuzzy max. Regarding the fuzzy sum, if A and B are TFNs, then the sum of A and B, $A + B$ is TFN as well [12]. Furthermore, the sum of two TFNs can be written as

$$(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \quad (1)$$

In this paper, the maximum ($\vee$) of two triangular fuzzy numbers is approximated by a triangular fuzzy number using the following equation:

$$(a_1, a_2, a_3) \vee (b_1, b_2, b_3) \cong (a_1 \vee b_1, a_2 \vee b_2, a_3 \vee b_3) \quad (2)$$

We can use this approximation because we work with the fuzzy interval rather than the membership function. Also, degree of the approximation depends on the overlap of the two triangular fuzzy numbers. In some cases, the results obtained by the above equation are exact rather than approximated.