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On the location of new facilities for chain expansion under delivered pricing

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ABSTRACT

We study the problem of locating new facilities for one expanding chain which competes for demand in spatially separated markets where all competing chains use delivered pricing. A new network location model is formulated for profit maximization of the expanding chain assuming that equilibrium prices are set in each market. The cannibalization effect caused by the entrance of the new facilities is integrated in the objective function as a cost to be paid by the expanding chain to the cannibalized facilities. It is shown that the profit of the chain is maximized by locating the new facilities in a set of points which are nodes or iso-marginal delivered cost points (points on the network from which the marginal delivered cost equals the minimum marginal delivered cost from the existing facilities owned by the expanding chain). Then the location problem is reduced to a discrete optimization problem which is formulated as a mixed integer linear program. A sensitivity analysis respect to both the number of new facilities and the cannibalization cost is shown by using an illustrative example with data of the region of Murcia (Spain). Some conclusions are presented.

1. Introduction

The location of facilities is a major decision for a chain that competes for customers demand with other chains offering the same type of product. A variety of location models have been proposed to cope with this kind of problem (see for instance [1–3]). When the competing chains use delivered pricing, profit is strongly affected by both the location of their facilities and the price they set in each market area. For spatially separated markets and homogeneous products, the main feature of many of these location problems is that each chain monopolizes a group of markets once the locations of the facilities are fixed. Therefore, each chain sets the optimal price in each one of its monopolized markets.

Delivered prices are frequently used when the ratio of transportation cost to the total price paid by the customers is high, which has been observed in many markets [4]. With this price policy, once the facility locations are fixed, a Nash equilibria in price for the competing chains can be found under quite general conditions. Hoover [5] analyzed this price policy for the first time, considering each chain locates one facility, and concluded that the equilibrium market price of a chain with the lowest delivery cost is equal to the next lowest delivery cost. This result has been extended to a spatial duopoly on a compact subset of the plane [6] and to a network [7]. The existence of Nash equilibrium under delivered pricing has been recently reconsidered in [8]. As result of price competition, the competing chains will set the equilibrium prices, if they exist, once they know the location of their facilities. In such a case, the location-price decision problem for a chain under competitive delivered prices reduces to a location problem.

This kind of location problem has been mainly studied on a network location space within two frameworks, one considering that all competing chains decide on location, another considering that there is one entering chain which decides on location and competes with other chains which have their facility locations already fixed. The first is seen as a non-cooperative game, for which a node-optimality property and some location equilibrium results have been given when each chain opens one facility [7,9]. A procedure to find location equilibria when each chain opens more than one facility is shown in [10]. The second is seen as an optimization problem, which has been solved in discrete location space by integer linear programming formulations for fixed demand in [11] and variable demand in [12]. In network location space, a node-optimality property is shown and the problem is solved for variable demand by a mixed integer linear programming formulation in [13].

The aim of this paper is to study the location problem in a new framework in which the existence of fixed facilities owned by some competing chains operating in the network is considered and one of such chains wants to expand by locating new facilities on the network. The new facilities will compete with each other, as well as with any existing facility, owned by the expanding chain or by any of its competitors. Therefore, the pre-existing facilities owned by the expanding chain can lose profit as a

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consequence of the expansion. This effect is known as *cannibalization* and it was first considered in franchise distribution systems (see [14,15]), but it has been almost ignored in the recent location literature. To our knowledge, cannibalization has been taken into account mostly in Huff-like location models, but it has not been studied under delivered pricing. In discrete location, it has been considered in a single-objective location model with variable expenditure functions in [16]. In that model the effect is not explicitly present, but the cannibalized facilities may increase their demand due to market expansion as result of the entrance of new facilities. In [17], a model to simultaneously optimize the locations and designs of a set of new facilities is studied in which the cannibalization effect is captured. In planar location, it has been considered as a secondary objective in multi-objective location of a single facility in [18,19], where lexicographic approaches are used. In this paper, a sharing profit model is formulated where the cannibalization effect is integrated in the objective function as a cost instead of as a secondary objective.

Our contribution is the formulation and study of a new location model under delivered pricing in which the profit lost by the existing facilities owned by the chain as result of the expansion is compensated by side payments. The income of the entire chain is a portion of the profit of its facilities which is determined under the assumption that the competing chains will set the equilibrium prices in each market once the new facilities are fixed. Minimum profit constraints are considered to make the new facilities economical to operate. We formulate this sharing profit model on a network location space where nodes and any point on the edges are location candidates. First we show that the profit of the chain (income minus side payments) is maximized by locating the new facilities in a finite set of points given by the nodes and the points in the network from which the marginal delivered cost equals the minimum marginal delivered cost from the existing facilities owned by the chain. Then the location model is reduced to a discrete optimization problem which is formulated as a mixed integer linear programming problem. Finally, the cannibalization effect caused by the new facilities is analyzed by using an illustrative example with real data. A sensitivity analysis with respect to the number of new facilities and the cost of side payment is also carried out.

In Section 2, basic hypothesis and notation are given, the equilibrium prices are determined, and the location model is formulated. In Section 3, the results concerning the maximization of the net profit are shown. In Section 4, the location model is reduced to a discrete optimization problem which is formulated as a mixed integer linear programming problem that can be solved by standard optimizers.

In Section 5, an illustrative example with data of the region of Murcia (Spain) is analyzed. Finally, some conclusions are presented in Section 6.

2. Preliminaries

Let $N(V,E,\ell)$ be an undirected network with node set $V=\{v_k : k=1,\dots,n\}$ and edge set $E=\{e : e=[v_k,v_j];\ v_k,v_j \in V\}$ on which the function ℓ is defined to be the length of edge e. Denote N as all points on the network (nodes and points on the edges). A generic point on the network, either a node or any point along an edge, is denoted by x. The distance between two points in N is defined by the length of the shortest path joining those points.

Let $M=\{1,\dots,m\}$ be a set of spatially separated market areas, $m \le n$, so that customers in market area k are assumed to be aggregated at node v_k (see [20] for demand point aggregation). We note that the network may contain some nodes on which no market is aggregated, which occurs if there are some linking nodes with no customers around, $m<n$. The customers demand a homogeneous product and they are served from some existing facilities which are located at some given nodes. These facilities are owned by different chains. Without loss of generality we can consider two chains: an expanding chain A, which wants to locate some new facilities, and its competitors, which are named as chain B. The competitors are supposed not to react by locating other new facilities, but they can change their prices after the expansion of chain A.

The two chains compete with delivered pricing, which means that each chain offers a price in each market, pays for the transportation cost, and delivers the product to the customers. Customers buy from the chain that offers the lowest price in the market they belong to. If the two chains offer them the same price, customers are indifferent to chain choosing. However, the chain with the minimum marginal delivered cost (production +transportation) can offer a lower price and gets the customers demand. Then, ties in price are broken in favor of the chain with the minimum marginal delivered cost.

We consider that the demand function in each market may be different from the demand function in other markets. Marginal delivered costs at each facility are supposed to be independent of the amounts delivered from the facility and the chains use linear prices. We also consider that the chains cannot sell the product at a price below their marginal delivered costs.

The decision variables for the expanding chain A are the set of locations for its new facilities and the set of prices to be set in the market areas after the expansion. The objective is profit maximization of the entire chain, but the cannibalization effect due to the expansion will also be taken into account.

The following notation is used:

Indices

k	index of demand nodes, $k=1,\dots,m$

Data

$M=\{1,2,\dots,m\}$	set of markets
$q_k(p)$	demand function in market k, which depends on selling price p
F_A	set of locations of existing facilities owned by chain A
F_B	set of locations of existing facilities owned by chain B
N	set of location candidates for the new facilities
r	number of new facilities to be located
c_x	marginal production cost at location x, $x \in N$
t_{xk}	marginal transportation cost from location x to market k
$C_{xk}=c_x+t_{xk}$	marginal delivered cost (or minimum delivered price) from location x to market k

Decision variables

X	set of locations for the new facilities
p_k	price the expanding chain sets in market k

Miscellaneous

d_{xk}	distance between location x and demand point k
$C_k(F)=\min\{C_{fk} : f \in F\}$	minimum delivered cost (or minimum selling price) from facilities in F to market k, $F \subset \{F_A \cup F_B \cup N\}$

2.1. Price competition

In this subsection, we briefly present some of the results shown in a previous paper [13] which are used to introduce the new location model.