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A Job Shop Scheduling Heuristic for Varying Reward Structures

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Abstract—In this paper, a job shop scheduling problem of assigning preemptive tasks with precedence constraints to flexible resources is defined. A significant characteristic of this problem is that each task has a reward structure that defines the amount of reward based on its completion time. The objective is to maximize the total reward in the job shop. An optimal solution to this problem cannot be realistically obtained, as even significantly simpler problems are strongly NP-hard; therefore, a heuristic is proposed to obtain applicable solutions. The method is based on calculating (heuristic) penalty indices for each task as a function of time in order to allocate tasks to resources. A mathematical programming technique is then used to optimally minimize the penalty. This approach combines sound management science principles in order to emphasize analytical solutions to complicated real-life problems.

Keywords—Job shop, Scheduling, Reward, Penalty index, Heuristics, Resource allocation.

1. INTRODUCTION AND LITERATURE REVIEW

Job shop scheduling is one of the most popular problem domains of management science. A review of literature provides numerous studies in this field, most of which are concerned with obtaining optimum or near-optimum solutions by defining strict assumptions that limit their practical usage. Wein and Chevalier [1] explain how most of these studies approach the problem in a narrow focus and those that address multiple objectives are restricted to the static and deterministic single-machine case. Baker [2] investigated recent trends in scheduling and emphasized the importance of heuristics: "since most realistically sized optimal scheduling algorithms are NP-complete, it makes sense to try and redefine the problem in a way that it can at least usually be solved". Although

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optimal solutions are usually the target of such relaxation, other attempts relax the requirement that the solution be optimal and aim to achieve a solution within some range of optimal. In these near-optimal algorithms, the range of optimality has been defined as how close the solution is to the optimal solution, how close the solution is to a bound on the optimal solution, or by probabilistic descriptions. The literature consists of various near optimal scheduling algorithms; however, none of the current methods appear to be a promising technique for the job shop scheduling problem considered in this paper.

The concern of this paper is to provide an efficient scheduling method for job shops with multiple jobs with varying reward structures, which are not static values, but a nonincreasing function of time (varying reward structures will be explained in the next section). These job shops generally have a mixture of flexible and static resources and deal with tasks varying in batch size, total process time and reward. In this context, *task* broadens the definition of *job*, which usually implies a set of processes with common characteristics. Here, a *task* may include any *operation* (e.g., process, design, transport) that requires a *resource* (e.g., machine, engineer, agv). Heuristic scheduling techniques are used to attempt to provide solutions with good performance and that are practical for industrial application. Moreover, the majority of the literature deals with job shop related objectives such as minimizing completion time, makespan or maximum tardiness, whereas the broader objective of any company is to maximize rewards.

Previous work in scheduling does not possess extensive research regarding reward-based problems. Pinedo [3] presents a collection of optimal algorithms for single machine, parallel machine, flow shop, open shop and job shop models from the literature. His widely accepted definition of a scheduling problem contains the triplet $\alpha \mid \beta \mid \gamma$, where α describes the processor environment and contains a single entry, β provides details of processing characteristics and constraints, and γ contains the objective function.

One problem that we can observe under the *reward* context is $1 \parallel \Sigma w_j(1 - e^{-rC_j})$, which has been solved optimally for the single-machine case using the *weighted discounted shortest processing time* (WDSPT) rule [4]. In this problem, costs are discounted at a rate of r, $0 < r < 1$, per unit time; that is, if job j is not completed by time t, an additional cost $w_j r e^{-t_j}$ is incurred over the period $[t, t + dt]$. If job j is completed at time t, the total cost incurred over the period $[0, t]$ is $w_j(1 - e^{-rt})$. The *weight* of each job, w_j, assigns a relative level of importance. The *apparent tardiness cost* (ATC) heuristic [5] is a composite dispatching rule for the problem $1 \parallel \min \sum w_j T_j$, where T_j is the tardiness of job j. In this problem, the cost is zero for $t \leqslant d_j$ and $w_j(t - d_j)$ for $t > d_j$, where d_j is the due date of job j. The measure *weight* is similar to the idea behind *reward*, however, *weight* is a static value for each job, whereas *reward* is a function of time and its structure varies with each task. Another similar objective assumes that there is a time-dependent cost associated with tardiness. This problem is different from minimizing total tardiness, which only considers the cost of tardiness of a job, not the amount of tardiness. Rothkopf and Smith [4] showed that there are two priority index rules to achieve this objective optimally on a single machine; however, a similar approach to the job shop has not been studied. The most similar study to our problem is a methodology with the objective of maximizing reward without preemptions that translates the job shop scheduling problem into a multidimensional knapsack problem, which is another NP-hard problem [6]. However, even an efficient method to solve the multidimensional knapsack problem would lack applicability as a scheduling method.

Thus, in this paper, a heuristic is proposed to obtain applicable solutions, based on calculating penalty indices for each task as a function of time and allocating tasks to resources by minimizing total penalty. The remainder of the paper is organized as follows. Section 2 provides problem description, including definition of the reward function. Section 3 presents the methodology used to create the penalty index function and resource allocation model, followed by an example and benchmarking. Finally, conclusions are presented in Section 4.